mathematical girls

story by Hiroshi Yuki　comic by Mika Hisaka

上

数学×青春
が描く
無限の旅!!

理系男子最強の萌え――
理系男子絶賛の数学小説、待望のコミカライズ

[原作]結城 浩
[作画]日坂水柯

数学ガール

MFコミックス

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数学×青春

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上

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メディアファクトリー

「僕」が憧れる女の子、
数学を何より愛するミルカさん。
彼女は、「僕」をどう思ってるんだろう。
「僕」を慕う女の子、後輩のテトラちゃん。
彼女は、「僕」に何を伝えたいのだろう。
数学が紡いでいく論理と、
ほのかでとらえられない気持ち。
数学の論理展開を学びながら、
数式では解けない恋の行く末を描く、
世界初の数学青春ストーリー登場!

「虚数単位」以外のアイなんてあるの?」
――あるよ、だから、僕は……

数学を愛するミルカさん。彼女に憧れる「僕」。
「僕」を慕ってくる後輩テトラちゃん。
3人が描く単位円の軌跡は、やがて……。

月刊コミックガーデル(毎月5日発売)にて好評連載中!

結城 浩

1963年生まれ。プログラム書きと文章書きが趣味＆仕事。著書は『プログラマの数学』『増補改訂版Java言語で学ぶデザインパターン入門』『新版暗号技術入門』など。パッハのフーガが好きなプロテスタントのクリスチャン。

http://www.hyuki.com/

日坂水柯

6月生まれA型。好きなTV番組はＮＨＫ高校講座地学

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数学ガール

上

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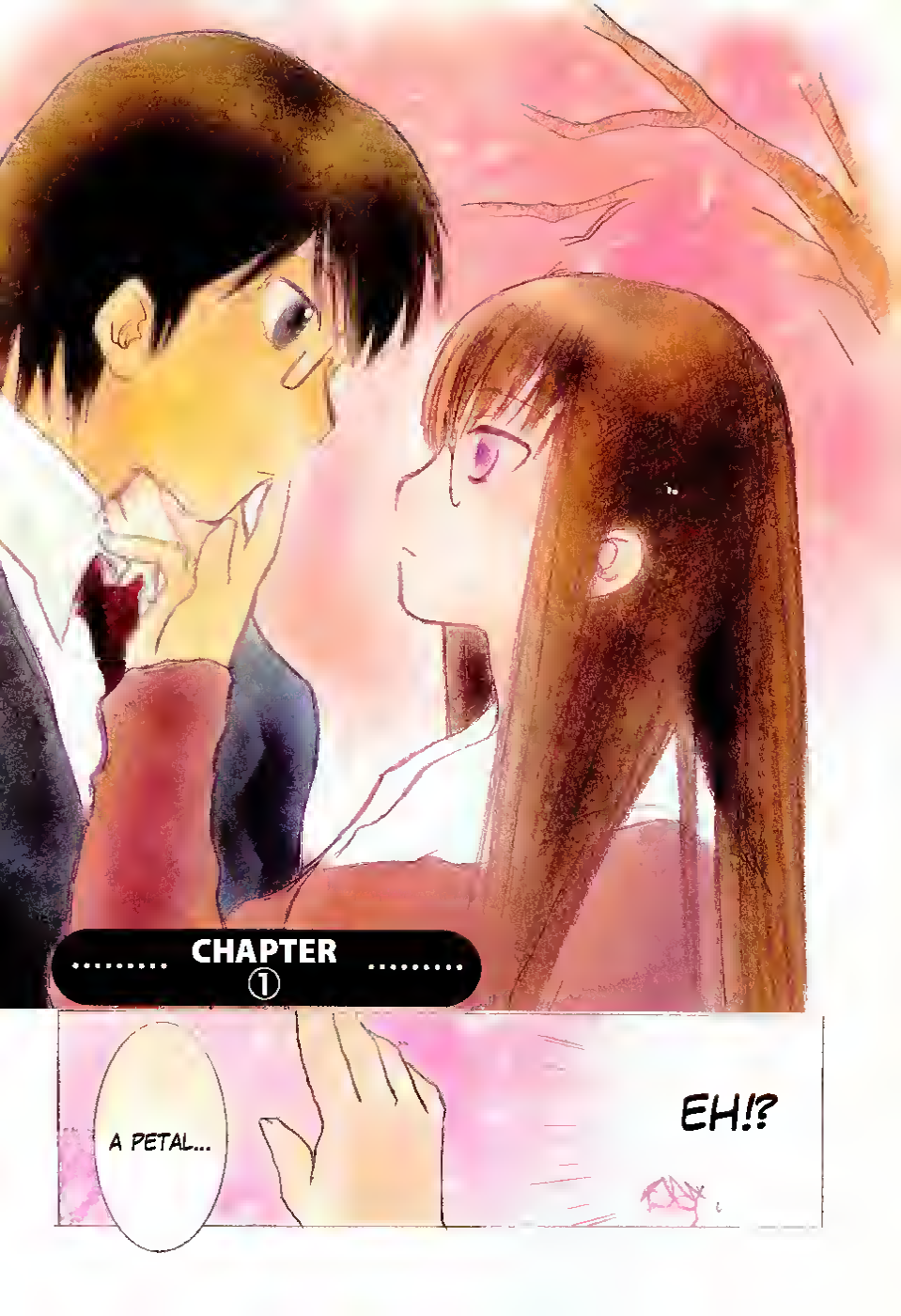
CHAPTER
①
A PETAL...
EH!?

I...
THERE'S A PETAL...
ON YOUR LIPS...
EVER SINCE THEN...

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この物語はフィクションであり
実在する人物・団体とは関係ありません

[原作]結城 浩
(『数学ガール』ソフトバンククリエイティブ)
[作画]日坂水柯

数学ガール
Mathematical Girls 上

story by Hiroshi Yuki comic by Mika Hisaka

Translated & Edited by: EdMX
WHAT IS IT?
UMM...
I WAS JUST WONDERING WHAT YOU'RE LOOKING AT...
YOUR FORMULAS.
WELL, YEAH I CAN SEE THAT, BUT...
I LIKE MATHEMATICS.

IT HAS BEEN A HABIT OF MINE SINCE JUNIOR HIGH
MAKING FORMULAS...
LIBRA
THAT'S WHY I'M ALWAYS AT THE LIBRARY AFTER SCHOOL
AND ABOVE ALL ELSE, I LIKE MAKING FORMULAS IN PARTICULAR.
I CAN'T SEE...
HOWEVER,
THAT PLAN OF MINE IS NOW...
I HOPED TO CONTINUE DOING SO, ALONE IN THE LIBRARY...
WHICH IS WHY COME HIGH SCHOOL,
I CAN'T SEE
YOUR CALCULATIONS.
WH-
WHAT?

IN MIRUKA'S CASE SHE'S...
HOW SHOULD I PUT IT... NOT WHAT SHE SEEMS? OR BETTER YET...
UNLIKE OTHER PEOPLE... OR MAYBE IT'S...
THERE'S A FEELING OF DETACH-MENT...
IT'S THE NUMBER SEQUENCE PUZZLES.
1, 1, 2, 3,
1, 4, 27, 256,
6, 15, 35, 77,
6, 2, 8, 2, 10, 18,

I PRESENT A PROBLEM.
ONE YEAR AGO...
1ST YEAR HIGH SCHOOL SPRING.
THAT WAS WHEN I FIRST MET HER.
BACK THEN, ALL SHE DID WAS...
first confrontation
1, 1, 2, 3, ...
HUH?
that came out of nowhere...
SPEAK IN NUMBERS...
SO I HUMORED HER...
next would be...
5, 8, 13...
HEY, ISN'T THAT...
1, 4, 27, 256, 3125, 46656, ...
WTH!?
tsk tsk
THEN...
THERE'S NO WAY I CAN CALCULATE IT MENTALLY!!
UH... NEXT WOULD BE...
3125
second confrontation
...
1, 4, 27, 256,
ENDED UP ANSWERING IN NUMBERS
EVEN I...
UHH...
really?
MISCAL-CULATION.
...
third confrontation
6, 15, 35, 77,
NEXT IS...
133!!
...
...

YOU'RE TURNING RED...
EH!?
AS FOR THE LAST ONE
THAT ISN'T EVEN A SEQUENCE.
AH-
ANYHOW...

...
MEH...
I GUESS YOU COULD SAY THAT...
BUT STILL, YOU WERE ABLE TO SOLVE IT.
YOU'RE RED UP TO YOUR EARS.
pat
WHA-
WHAT ARE YOU-
BUT EVEN THOUGH THAT WAS SUPPOSEDLY THE FIRST TIME I MET HER, HOW DID SHE FIGURE OUT I LIKED MATHEMATICS?...
they're cool...
I know!
they're sta—ring at us...
.....

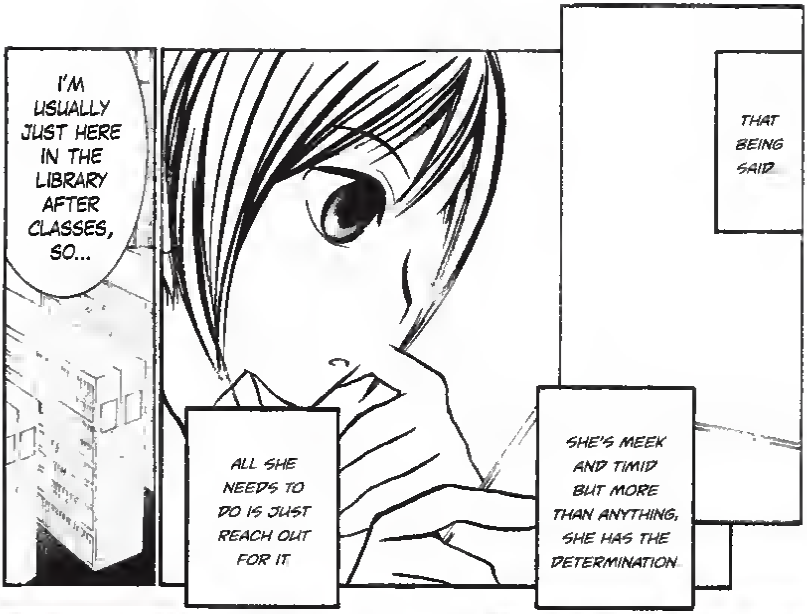
HAVE YOU EVER HEARD OF THE "NUMBER SEQUENCE WITH NO ANSWER"?
HOW DOES THAT WORK?
FOR EXAMPLE...
1, 2, 3, 4, ...
WELL?
IT SHOULD BE 5.
IN THIS SEQUENCE, WHAT DO YOU THINK COMES NEXT?
1, 2, 3, 4, 5, ...

1, 2, 3, 4,
WHAT IF, SAY...
1, 2, 3, 4, 10, 20, 30, 40
IT WERE TO SUDDENLY INCREASE, LIKE SO...
1, 2, 3, 4, 10, 20, 30, 40, 100, 200, 300, 400,...
AND EVEN MORE...
IN OTHER WORDS, HOW MANY NUMBERS MUST WE SEE BEFORE WE CAN TRULY DEDUCE ITS SEQUENCE?
SOMEHOW, THAT'S KINDA UNFAIR..
IN THIS CASE, SUCH A SEQUENCE IS POSSIBLE.
AND YOUR POINT IS?
BUT FROM THIS POINT ONWARD, WE SEE THAT THE VALUE MUST INCREASE, ABRUPTLY.
HERE, WE WERE ONLY GIVEN THE FIRST FOUR NUMBERS OF THIS SEQUENCE,
RIGHT?
NOT NECESSARILY SO...

IF A NUMBER SEQUENCE EXTENDS TOWARDS INFINITY,
HOW MANY NUMBERS MUST BE KNOWN IN ORDER TO DETERMINE THE REST?
THIS IS WHAT IS MEANT BY A "NUMBER SEQUENCE WITH NO ANSWER".
PRESENTED AS IT IS, THERE IS STILL THE POSSIBILITY OF A DRAMATIC CHANGE IN THE SEQUENCE OF PATTERNS THEMSELVES.
BUT FOR THE SEQUENCE OF
1, 2, 3, 4,
TO SUDDENLY JUMP TO
10...
WHERE IS THE PROBLEM, THERE?
PEOPLE, IN GENERAL, WOULDN'T PERCEIVE THAT AS A PROBLEM.

LET GO OF ALL ASSUMPTIONS...
BECAUSE YOU WILL NEVER KNOW WHAT NUMBER WILL COME NEXT IN THE SEQUENCE.
ずばっ
...
watch where you're going
yikes
1, 2, 3, 4, 6, 9, 8, 12, 18, 27, ...
That aside, do you understand the expression used in this sequence?
Hmm. It's like I get it. And then I don't.
1 2 3 4 6, 9 You'd expect that the following number would be larger, right?
HER DISCUSSIONS ABOUT MATHEMATICS ARE DIFFICULT TO UNDERSTAND, AND YET...
IT'S SO INTERESTINGLY STIMULATING.
Hmm... Disregarding the first 1 in the sequence, all the numbers are either multiples of 2 or 3. But it's strange that the number suddenly decreases like that.
But that is not the case. What comes after 9 is actually 8. Do you note the pattern here?
What if I told you that it's answer is wrong...
COMPARED TO ME,
MIZUKA'S STRANGE SENSE OF DETACHMENT...
AND HER EASY-GOING ATTITUDE, UNAFFECTED BY WHAT OTHERS THINK...

I COULD NEVER COME TO HATE.
THAT IS SOMETHING...
After class...
LIBRARY
DO YOU GET IT?
THIS IS TETORA-CHAN, A JUNIOR OF MINE
I TUTOR HER IN MATHEMATICS
.....
I GET IT!!
I...
I THINK...


I'M USUALLY JUST HERE IN THE LIBRARY AFTER CLASSES, SO...
THAT BEING SAID
ALL SHE NEEDS TO DO IS JUST REACH OUT FOR IT
SHE'S MEEK AND TIMID BUT MORE THAN ANYTHING, SHE HAS THE DETERMINATION

"Are you and that girl...
UH...
UMM...
There is something I want to ask...
IF YOU HAVE ANY QUESTIONS, FEEL FREE TO STOP BY AND ASK AWAY

...are the two of you going out?*
HMM?
I CAN'T... SAY IT!
ah! forget it!
OH! IT- IT'S NOTHING, IT'S NOTHING!...
sfft

EH!?
EEH!?
WHA-!?
From back then...
IT GOT CAUGHT IN YOUR HAIR.
Right here, near your nape.
EEEEEH!?
There's no way he'd touch me in that kind of way...
THERE WE GO.
...but still

has my heart racing...
UWAWAWA!
The faint warmth of senpai's touch...
yet again.
ONE YEAR AGO...
MIRUKA PRESENTED ME WITH FOUR NUMBER SEQUENCE PUZZLE
1、1、2、3、5、8、13、21、…
THIS IS THE FIBONACCI SEQUENCE.
STARTING WITH 1, 1, THE SUCCEEDING NUMBER IS THE SUM OF THE PREVIOUS TWO NUMBERS

6、15、35、77、143、…

2×3　3×5　5×7　7×11　11×13

To think that I miscalculated 11x13... that kinda hurt my pride

SIMPLY PUT, THIS SEQUENCE FOLLOWS THE **PRODUCT OF SUCCESSIVE PRIME NUMBERS.**

1、4、27、256、3125、46656、……

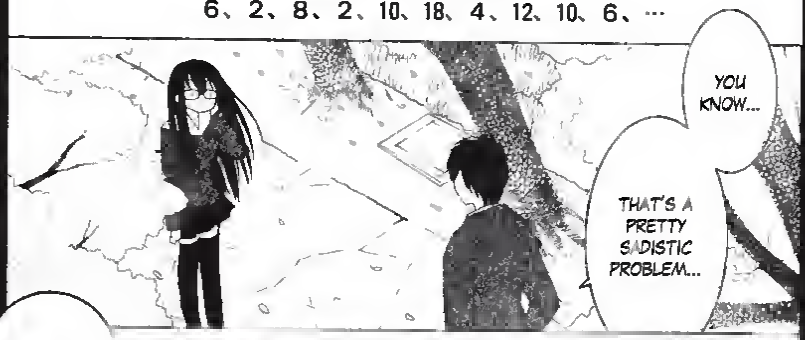
1^1、2^2、3^3、4^4、5^5、…　FOR THIS SEQUENCE...

That's why I said it's impossible because you just can't calculate 6^6 in your head, I done.

IT FOLLOWS THE USE OF THE EXPRESSION n^n

The final problem

6、2、8、2、10、18、4、12、10、6、…



YOU THINK SO?

THIS IS AN EXPANSION OF THE DECIMAL PLACES OF THE VALUE PI, MULTIPLIED BY 2 AS FOLLOWS

$\pi = 3.141592653\cdots\cdots$

→3	1	4	1	5	9	2	6	5	3、…
(×2)	↓	↓	↓	↓	↓	↓	↓	↓	↓	
→6	2	8	2	10	18	4	12	10	6、…

I GUESS...
YOU COULD SAY THAT...
sfft

A PETAL...
THERE'S A PETAL ON YOUR LIPS.
EH!?

I...
EVER SINCE THEN...

I COULDN'T THINK OF ANYTHING BUT HER.
Mathematical Girls Chi
Translated & Edited by:
EdMX

T I P S

[The Fibonacci Sequence]

This number sequence was named after the Italian mathematician, Leonardo Fibonacci. The definition for deriving this sequence was explained, earlier, in the manga. The Fibonacci Sequence is present in a great number of natural phenomena. For example, it is well known that if you count the number of spines on a pine cone, chances are you would arrive at a Fibonacci Sequence number.

数学ガール

Mathematical Girls

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LIBRARY
MENTAL MATH CHALLENGE.
HOW MANY FACTORS ARE THERE IN THE NUMBER 1,024?
JUST MENTALLY?
I'M GIVING YOU TILL THE COUNT OF 10 TO ANSWER
ZERO.
ONE.
TWO...
I've heard that question somewhere before...
Translated & Edited by: MX

LET'S SEE...
1 AND 2 ARE PROBABLY FACTORS...
3 WON'T WORK...
IT LOOKS LIKE IT'S DIVISIBLE BY 4..
FOUR.
FIVE.
SIX.
AH!
1,024 IS ALSO 2^0!
AND HOW WERE YOU ABLE TO SOLVE THAT, MENTALLY?
BY USING PRIME FACTOR DECOM-POSITION, YOU'LL SEE THAT 1,024 IS MADE UP OF 10 NUMBER 2'S.
ELEVEN.
...NINE.
TEN.
THE NUMBER 1,024 HAS ELEVEN FACTORS.
YOUR ANSWER?
$1024 = 2^{10} = 2\times2\times2\times2\times2\times2\times2\times2\times2\times2$
10 number 2's
SIMPLY PUT, IT WOULD LOOK LIKE THIS.

IN SHORT...
THEREFORE, THE FACTORS MUST FOLLOW THE EXPRESSION 2^n WHERE 'N' IS A VALUE FROM 0 TO 10.
ALL OF THE FACTORS NEED TO BE ABLE TO DIVIDE 1,024.
I've even heard the answer before, too...
2^0, 2^1, 2^2, 2^3, 2^4, 2^5, 2^6, 2^7, 2^8, 2^9, 2^10
THE FACTORS OF 1,024 ARE THESE ELEVEN VALUES.
But...
HMM... YOU GOT IT.
What in the world is he talking about?
I seriously can't keep up with what he's saying...
CHAPTER ②

I used to think that arithmetic problems were interesting...

Because back when I was still in elementary school,

It looks like senpai is really enjoying himself.

I can totally understand why, too.

Reference Book A

$(x+y)(x-y)=x^2-y^2$

Reference Book B

$(a+b)(a-b)=a^2-b^2$

The moment I entered junior high, however...

It began to feel like math was becoming my waterloo...

I wasn't expecting that at all...

However, come exam day, all I got was a big fat F.

So I thought that if I focused on the scope of the midterms, the test would pretty much answer itself.

There was no way I was going to let myself flunk the exams.

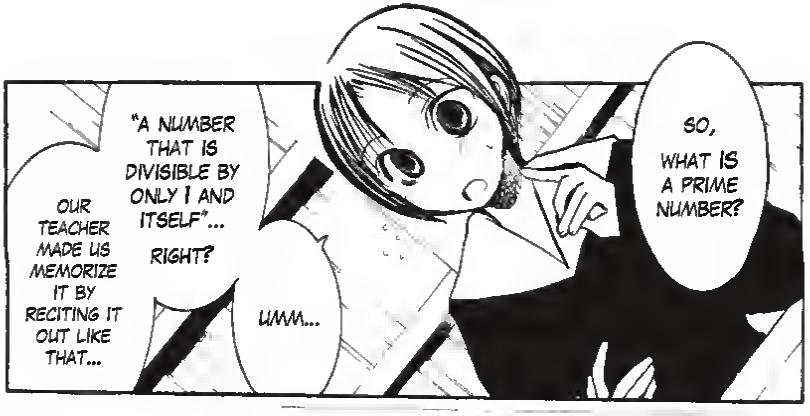
...

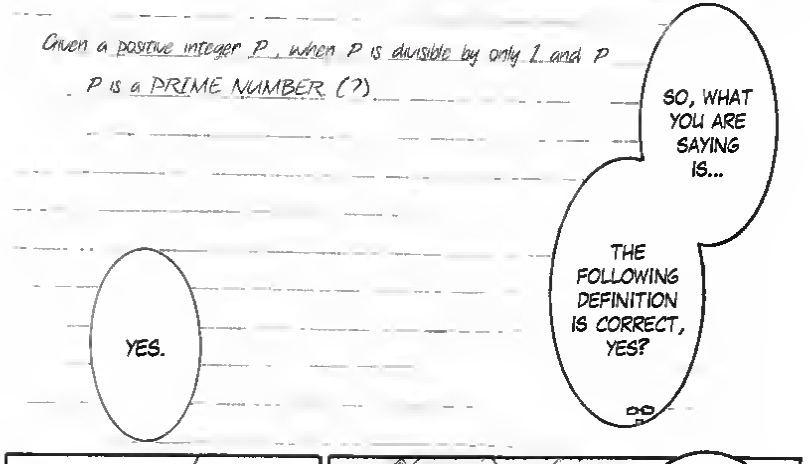
Senpai...

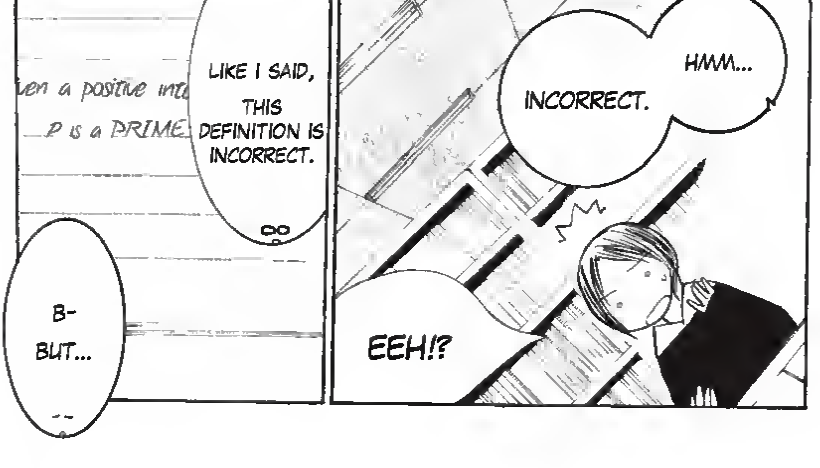
I heard that come high school, you're expected to be good in mathematics already...

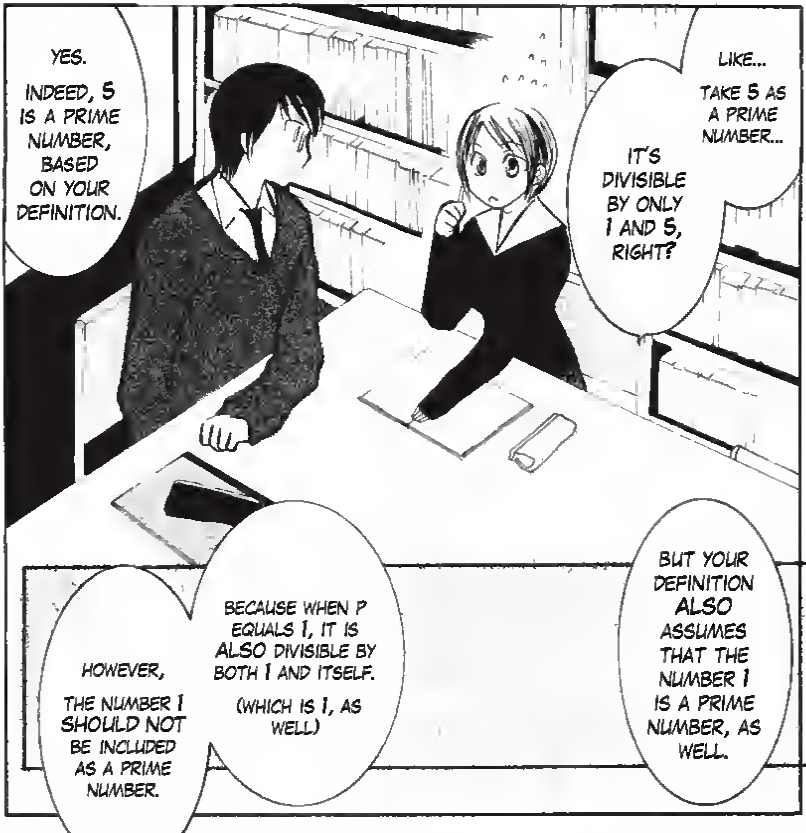
Which is why,
I consulted with an upperclassman who is really good in math.
Senpai...
He's one year above me and went to the same junior high as me.
WELL... I THINK I DO... THAT IS...
DO YOU UNDERSTAND THIS TOPIC?
WHEN IT COMES TO MATH PROBLEMS OR THE CLASS ITSELF, YOU'RE ALWAYS SO NEUTRAL ABOUT HOW YOUR DOING
BUT IN REALITY, YOU JUST DON'T GET IT, DO YOU?
BUT I DO!
TRUTH IS, YOU THINK YOU DON'T.

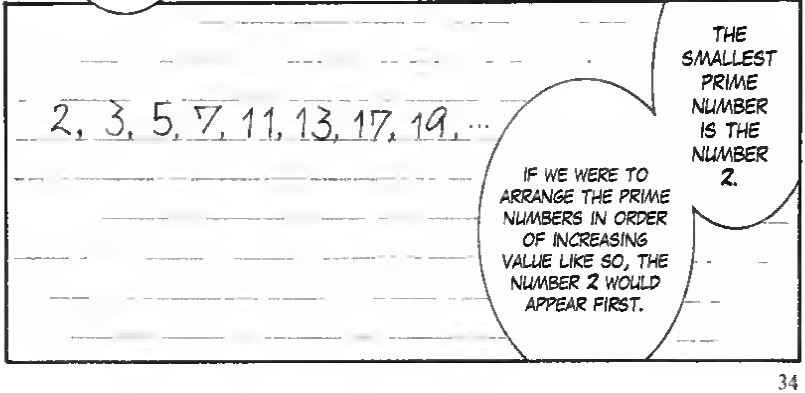
HUH?
SO TELL ME...
DO YOU KNOW WHAT A PRIME NUMBER IS?
Y- YEAH.
IN OTHER WORDS, I WANT YOU TO TELL ME WHAT A PRIME NUMBER IS.
EVEN IF IT'S IN YOUR OWN WORDS, IT'S FINE.
UHH..
HUH?
VERY WELL, THEN... PLEASE DEFINE A PRIME NUMBER.
AN EXAMPLE IS NOT THE SAME AS A DEFINITION.
OKAY.
TRUE... 5 AND 7 ARE BOTH PRIME NUMBERS...
BUT THOSE ARE JUST EXAMPLES OF WHAT PRIME NUMBERS ARE.
YOU MEAN LIKE... 5 AND 7 LIKE THAT?


SO, WHAT IS A PRIME NUMBER?
UMM...
"A NUMBER THAT IS DIVISIBLE BY ONLY 1 AND ITSELF"... RIGHT?
OUR TEACHER MADE US MEMORIZE IT BY RECITING IT OUT LIKE THAT...


Given a positive integer P, when P is divisible by only 1 and P
P is a PRIME NUMBER (?)
SO, WHAT YOU ARE SAYING IS...
THE FOLLOWING DEFINITION IS CORRECT, YES?
YES.


LIKE I SAID, THIS DEFINITION IS INCORRECT.
B- BUT...
HMM...
INCORRECT.
EEH!?

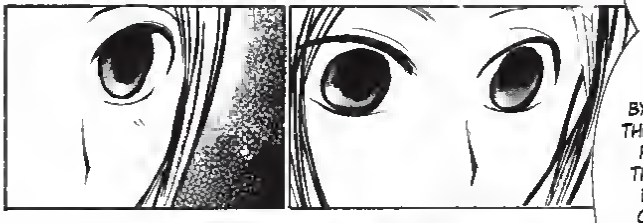

YES.
INDEED, 5 IS A PRIME NUMBER, BASED ON YOUR DEFINITION.
LIKE...
TAKE 5 AS A PRIME NUMBER...
IT'S DIVISIBLE BY ONLY 1 AND 5, RIGHT?
BUT YOUR DEFINITION ALSO ASSUMES THAT THE NUMBER 1 IS A PRIME NUMBER, AS WELL.
BECAUSE WHEN P EQUALS 1, IT IS ALSO DIVISIBLE BY BOTH 1 AND ITSELF.
(WHICH IS 1, AS WELL)
HOWEVER,
THE NUMBER 1 SHOULD NOT BE INCLUDED AS A PRIME NUMBER.


2, 3, 5, 7, 11, 13, 17, 19, ...
IF WE WERE TO ARRANGE THE PRIME NUMBERS IN ORDER OF INCREASING VALUE LIKE SO, THE NUMBER 2 WOULD APPEAR FIRST.
THE SMALLEST PRIME NUMBER IS THE NUMBER 2.

Given a positive integer P, when P is divisible by only 1 and P

P is a PRIME NUMBER (?)

THIS IS WHY YOUR DEFINITION IS INCORRECT.



BY ATTACHING THE FOLLOWING PROVISION, THE CORRECT DEFINITION OF A PRIME NUMBER BECOMES SUCH.

Given a positive integer P, when P is divisible by only 1 and P

P is a PRIME NUMBER, with the exception of 1.

YOU MAY ALSO OPT TO WRITE THE CONDITION AT THE START OF THE DEFINITION...

Given an integer P greater than 1,

when P is divisible by only 1 and P, P is a PRIME NUMBER.

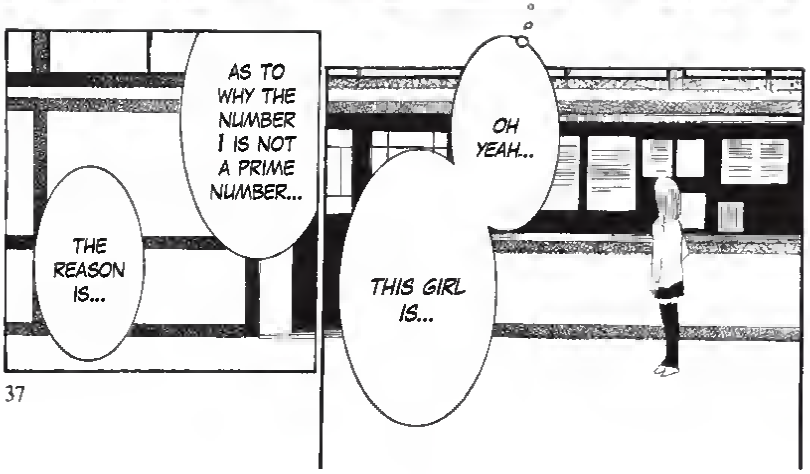
Given an integer P > 1, when P is divisible by only 1 and P

P is a PRIME NUMBER.

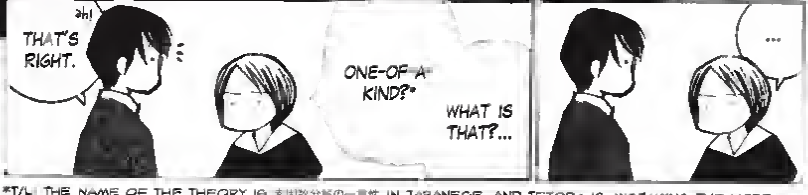
OR, YOU COULD WRITE THE CONDITION IN THE FORM OF AN EXPRESSION.

JUSTIFIED REASONING BASED ON ACCEPTED PRINCIPLES AND THEORIES AS PROVEN THROUGH LOGICAL PROOF!!
HOW-EVER...
I'M PRETTY SURE YOU HAVE LEARNED THIS BEFORE.
IT SHOULD BE CLEAR THAT THE NUMBER 1 MUST NOT BE INCLUDED AS A PRIME NUMBER.
WHY MUST WE NOT INCLUDE IT?
WHERE'S THE LOGIC IN THAT?
WHY ISN'T IT A PRIME NUMBER?
RATIONALE? ..
YES!
I DON'T UNDERSTAND THE RATIONALE IN NOT INCLUDING 1 AS A PRIME NUMBER...

UMM... SENPAI?
... UH- YEAH.

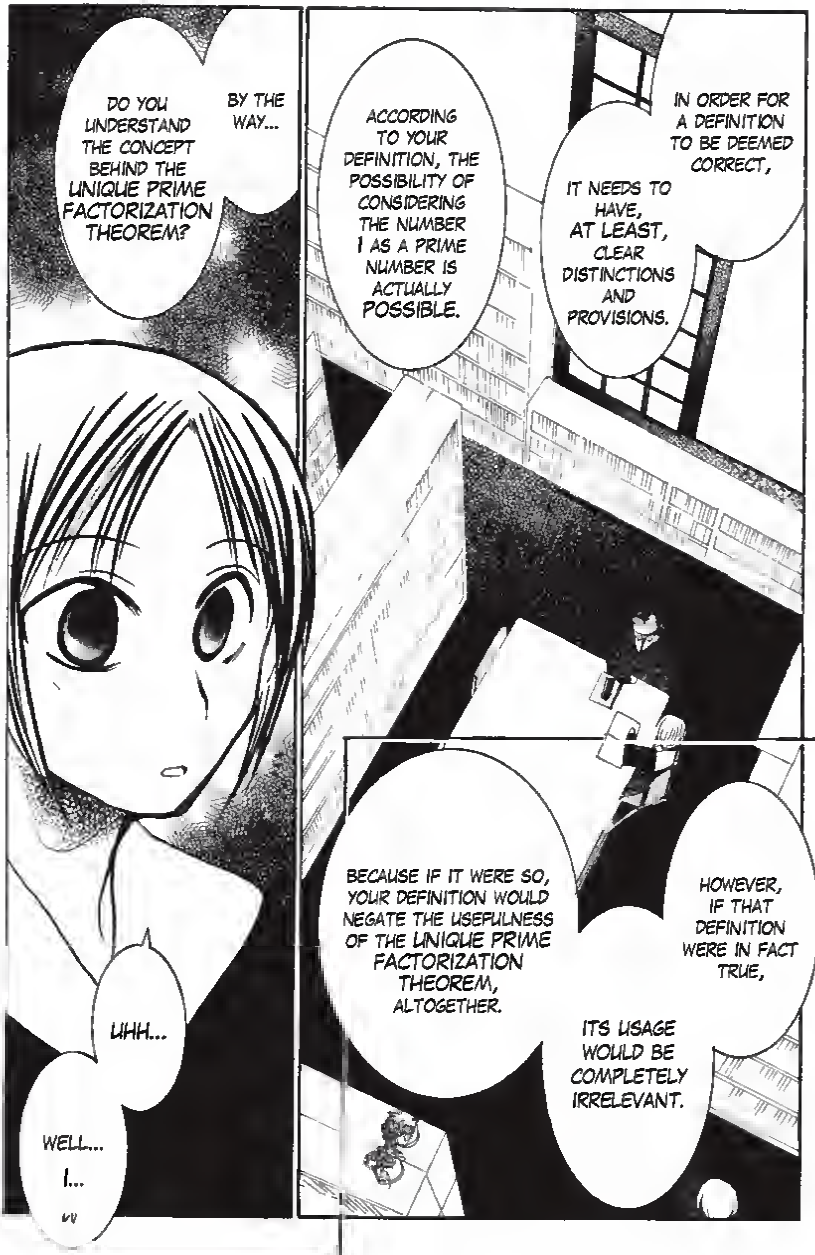

AS TO WHY THE NUMBER 1 IS NOT A PRIME NUMBER...
THE REASON IS...
OH YEAH..
THIS GIRL IS...

UNIQUE PRIME FACTORIZATION THEOREM

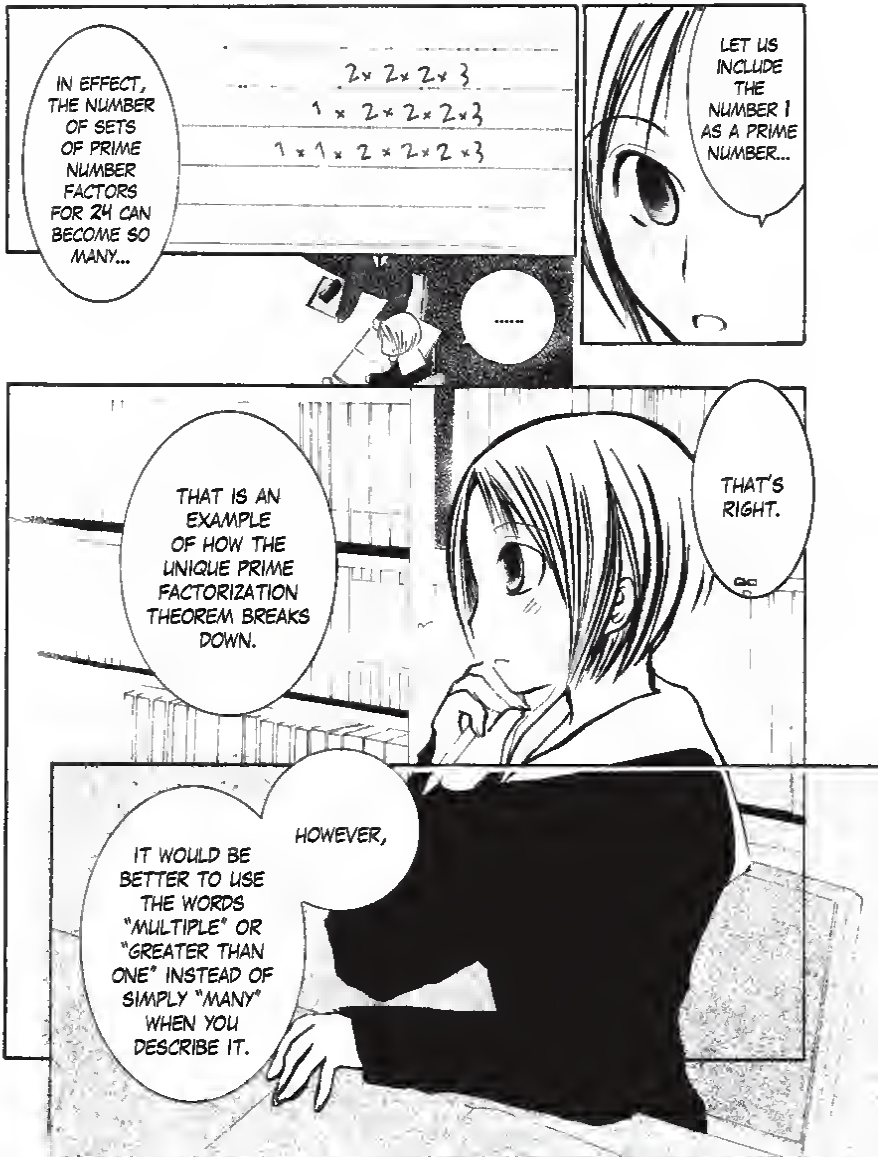


*T/L: THE NAME OF THE THEORY IS 素因数分解の一意性 IN JAPANESE, AND TETORA IS MISTAKING THE WORD 一意性 (ICHIISEI) LITERALLY TO MEAN "ONE-OF-A-KIND" INSTEAD OF "UNIQUE". SHE'S JUST NOT FAMILIAR WITH THE PROPER TERM ASCRIBED TO THE THEORY

FOR EXAMPLE, THERE IS ONLY ONE SET OF PRIME FACTORS FOR 24.
prime factors of 24
2 x 2 x 2 x 3
THE THEOREM STATES THAT ALL POSITIVE INTEGERS HAVE A SINGLE, UNIQUE SET OF PRIME NUMBER FACTORS.
THE UNIQUE PRIME FACTORIZATION THEOREM IS VERY IMPORTANT IN THE STUDY OF MATHEMATICS.
2 x 2 x 2 x 3
3 x 2 x 2 x 2
the prime factors are still the same
THE ORDERING OF THE NUMBERS DOES NOT MATTER.
IT IS IN OBSERVANCE OF THIS PROPERTY OF NUMBERS THAT THE NUMBER 1 MUST NOT BE INCLUDED AS A PRIME NUMBER, PER DEFINITION.
ONCE A MATHEMATICAL CONCEPT HAS BEEN DISCOVERED,
MATHEMATICIANS ASCRIBE SUCH NAMES FOR THE SAKE OF THE DEVELOPMENT OF THE SUBJECT, AS A WHOLE.
AND THAT, PRECISELY, IS WHAT A DEFINITION IS.
IT IS.
"IN ORDER TO FULFILL THE UNIQUE PRIME FACTORIZATION THEOREM"?
IS SUCH A CONVENIENT DEFINITION OKAY?
hmm
BUT SAYING IT'S "CONVENIENT" IS OVERSIMPLIFYING IT..
UHH...
.....


BY THE WAY...
DO YOU UNDERSTAND THE CONCEPT BEHIND THE UNIQUE PRIME FACTORIZATION THEOREM?
UHH...
WELL... I... uu
IN ORDER FOR A DEFINITION TO BE DEEMED CORRECT,
IT NEEDS TO HAVE, AT LEAST, CLEAR DISTINCTIONS AND PROVISIONS.
ACCORDING TO YOUR DEFINITION, THE POSSIBILITY OF CONSIDERING THE NUMBER 1 AS A PRIME NUMBER IS ACTUALLY POSSIBLE.
HOWEVER, IF THAT DEFINITION WERE IN FACT TRUE,
ITS USAGE WOULD BE COMPLETELY IRRELEVANT.
BECAUSE IF IT WERE SO, YOUR DEFINITION WOULD NEGATE THE USEFULNESS OF THE UNIQUE PRIME FACTORIZATION THEOREM, ALTOGETHER.

THE CONCEPT... IS...
YOU SHOULD REALLY TRY PROVING IT FOR YOURSELF, YOU KNOW.
BEFORE YOU END UP SAYING "I THINK",
I WILL TRY PROVE IT...
FOR MYSELF...
ALL RIGHT.
I THINK...
AN ILLUSTRATION IS NOT THE SAME AS A DEFINITION, HOWEVER...
ILLUSTRATING WITH THE USE OF PRACTICAL EXAMPLES IS GOOD PRACTICE, NONETHELESS.
LET'S USE SOME PRACTICAL EXAMPLES.
BUT EVEN THOUGH YOU SAY SO...
HOW AM I SUPPOSED TO DO THAT?


When 1 is considered a PRIME NUMBER, the UNIQUE PRIME FACTORIZATION THEOREM becomes NULL & VOID. EXAMPLES
2×2×2×3
1×2×2×2×3
1×1×2×2×2×3
IN EFFECT, THE NUMBER OF SETS OF PRIME NUMBER FACTORS FOR 24 CAN BECOME SO MANY...
.....
LET US INCLUDE THE NUMBER 1 AS A PRIME NUMBER...
THAT'S RIGHT.
THAT IS AN EXAMPLE OF HOW THE UNIQUE PRIME FACTORIZATION THEOREM BREAKS DOWN.
HOWEVER,
IT WOULD BE BETTER TO USE THE WORDS "MULTIPLE" OR "GREATER THAN ONE" INSTEAD OF SIMPLY "MANY" WHEN YOU DESCRIBE IT.

AND WHY DO YOU THINK IT'S BETTER TO USE THESE WORDS, INSTEAD?
SO THAT OUR DESCRIPTION WILL BE PRECISE...?
PRECISELY!
THE WORD "MANY" IS NOT A PRECISE TERM, AT ALL.
BECAUSE IT IS NOT CLEAR HOW MANY "MANY" IS WHEN YOU USE IT TO DESCRIBE.
The way things are, now...
DEFINI-TIONS.
PRIME NUMBERS
ILLUSTRA-TIONS
THE UNIQUE PRIME FACTORIZA-TION THEOREM.
ALL OF THEM.
THEY ALL USE PRECISE TERMS IN THEIR DEFINITIONS.
Having senpai come to teach me mathematics.
Finding myself coming back to the library every so often...

LANGUAGE IS VERY IMPORTANT IN THE STUDY OF MATHEMATICS.
LIKEWISE, YOU ARE CORRECT.
YOU ARE WISE TO HAVE NOTED THIS DISTINCTION.
AFTER ALL, MATHEMATICS EMBRACES LANGUAGE, JUST THE SAME.
IT ENJOYS THE USE OF SUCH PRECISE TERMS.
BY DOING SO, WE HELP PREVENT MISUNDERSTANDINGS FROM OCCURRING.
THAT SAID...
THE MOST PRECISE LANGUAGE TO EXPRESS MATH IS THROUGH MATHEMATICAL FORMULAS!

SENPAI...
YOU REALLY LOVE MATHEMATICS, DON'T YOU?
eh?
I... I GUESS SO!
There are still so many things I don't understand...
and many more things I have yet to understand...
However...
Even if that's the case, I just want senpai to-

KA-
CHAK!!

AFTERNOON-
CONTINUATION.
WHAT IS THE SUM OF THE DIVISORS OF 1,024?

TWO...
TWO THOUSAND FORTY-SEVEN...
Mathematical Girls Ch2
Translated & Edited by:
EdMX

T I P S

[Prime Numbers]

These are positive integers (whole numbers) that can only be divided by the number 1 and itself, with the exception of the number 1. For all positive integers, if the number 1 were considered a prime number, then the number of prime number factors for the said integer would be more than one. The existence of an infinitude of prime numbers was proven by the Greek Ancient, Euclid

GREETINGS!

EDMX HERE!

GUESS I OUGHT TO INTRODUCE MYSELF. I'M EDMX FROM GENESIS TRANSLATIONS. I RECENTLY WENT UP AND STARTED MY OWN "OFF-SHOOT" TRANSLATION BLOG FOR NON-H MATERIAL. AS A TRANSLATOR, I'VE ALWAYS BEEN OPEN TO TRANSLATING ALL SORTS OF THINGS, BUT IT'S GREAT TO RETURN BACK TO THE MORE MAINSTREAM WORKS.

TO START OFF IS THIS RATHER INTRIGUING MANGA, MATHEMATICAL GIRLS, WHICH A GOOD FRIEND OF MINE NAGGED TO HAVE TRANSLATED. SUFFICE IT TO SAY THAT HE'S A MATH-GENIUS, HIMSELF, SO I DIDN'T QUESTION WHY HE'D BE SO INTERESTED IN THIS PARTICULAR WORK.

SURPRISINGLY, THOUGH, IT'S QUITE INTRIGUING - AND IF IT WEREN'T FOR THE NOSEBLEED MOMENTS INVOLVED IN TRANSLATING THE THING, I'D SAY I'M QUITE EXCITED TO KNOW WHAT COMES NEXT. THAT SAID, LOOK FORWARD TO THE NEXT CHAPTERS, WHICH I HOPE TO RELEASE ON A REGULAR BASIS

UNTIL NEXT TIME, SEE YOU GUYS AROUND, AND DON'T FORGET TO CHECK OUT MY BLOG, AS WELL!

- EDMX

数学ガール

Mathematical Girls

DISCLAIMER:
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Though not listening,
I stood there wondering,
If that were the case,
Then even if she were to act like she just did...
If those two were dating or not.

Sorry about today.
I was just too shocked by it all
so I let it happen without saying anything
There wasn't anything I could really do about it...

Chapter 3

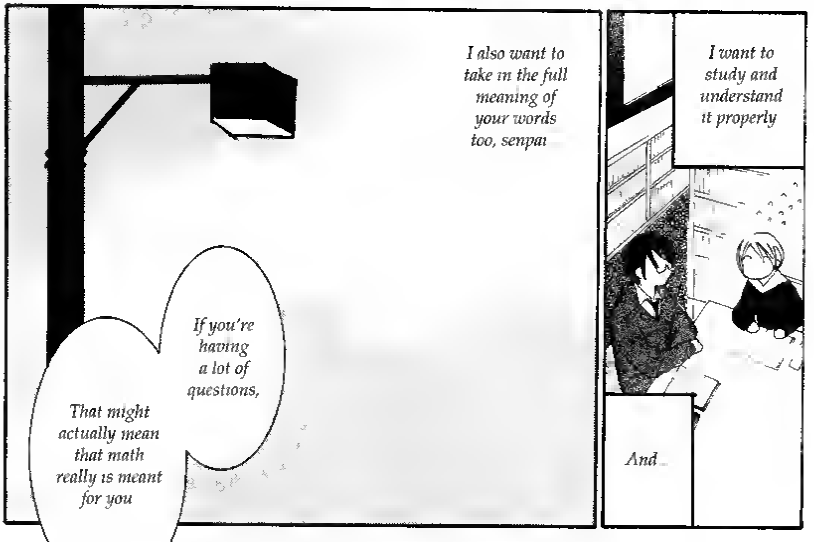
"Are they going out...?"
Y-yes
It sure was surprising, wasn't it?
Are you alright?

Umm...
Senpai...
I REALLY REALLY WANT TO ASK!!!

I wanted to ask about, uhh...
$(a+b)(a-b)=a^2-b^2$
This formula right here in the textbook...
Are you finding something about it troubling?
Y-yes
When I look at formulas like these,
I wonder using which variables is better, a and b or x and y?
I just can't bring myself to ask him...

I already take twice as long to do anything as it takes other people to do
But on top of that, I keep thinking about these questions
When I ask my friends who're good at math, they answer like
"You don't need to pay any attention to that."
"You really gotta pay more attention to stuff like this."
And just when I take their advice, at other times,
They'll answer like this
I can't seem to make any progress in my actual studies
And because I worry about such things like that everytime I look at formulas
Even when you give me your explanations, I'm not entirely sure how I should interpret it
While everyone else is wondering why I'm having so much trouble
I wrack my brain over what things I need or needn't worry about,

I'm still keenly interested in it.
I get the feeling that maybe math just isn't for me.
But even so...


I also want to take in the full meaning of your words too, senpai
I want to study and understand it properly
If you're having a lot of questions,
That might actually mean that math really is meant for you
And...

Do you think a lot about math as well?
Yup
But thinking thoroughly about these issues instead of passively taking them in is a critical process
It's true that it is quite time-consuming.
Like about what?
Let's see...
Here's your homework.

Homework assigned by Milka

Given a positive integer n, find a method to obtain the sum of all its divisors.

... Well... I think a lot about math problems...
and math formulae
Are all people who enjoy math like that?
I think so.
Don't you have something that you like to do as well, Tetra?
For instance, I'll try to reconstruct step-by-step formulae that I've encountered in class to see if I really understand the material or not. Or, I'll do example problems that I've done in class to see if I can solve them on my own

Ah-
...How did you know?
Because it's like that for me too.
Hm...
Well I do like English a lot
Then you must have English books in your bag, right?
And when you go to bookstores, you head for the English section first
Book
Ah! When did I
In my case, I head for the science and mathematics section first
When I stop by the usual bookstore I frequent,
I've memorized where all the books are placed in the science and mathematics section,
It doesn't matter which bookstore I go to, that's always how it is
So I can easily see which books are new just by glancing at the shelves

To solve this problem, I could just write down "figure out all the divisors of n and then add them up." But since that's a little boring and imprecise, I'll see if I can come up with a better solution. Hmm... Why don't I try prime factorization to break down the integer n? I solved that problem at lunchtime by realizing that $1024=2^{10}$. If I just generalized this method... For instance, let's choose to express n as a power of a prime number as shown below.

$n=p^m$ where p is a prime number and m is an integer

If $n=1024$, then according to the above equation, $p=2$ and $m=10$.

Writing all the divisors of n out would then yield the following.

$$1, p, p^2, p^3, \dots, p^m$$

Thus when $n=p^m$, the sum of all the divisors of n would be

Sum of all the divisors of $n = 1+p+p^2+p^3+\dots+p^m$

So now that I've successfully come up with a formula for integer n which can be expressed as $n=p^m$, I just need to generalize this a bit more... Hmm, this actually isn't all that difficult. I should first start out by expressing prime factorization in general terms.

Let base variables p, q, r, ... be prime numbers and exponent variables a, b, c, ... be positive integers. Then any positive integer n can be expressed as shown below.

$$n=p^a \times q^b \times r^c \times \dots$$

Wait a minute! I can't just use regular alphabets since the exponents a,b,c... will eventually reach p,q,r... which would confuse my terms. Just as in $2^3 \times 3^3 \times 7^4 \times \dots 13^3$, I have to come up with a way that shows precisely that all I'm doing is (prime number)$^{\text{positive number}}$.

Ah, I got it. I'll express the prime numbers as $p_0, p_1, p_2, \dots, p_m$ and the exponents as $a_0, a_1, a_2, \dots, a_m$. Writing all these subscripts might be a little cumbersome but I'll have to bear with it for now. Using this notation, m+1 will then be the number of all the prime number divisors that integer n has. Now then, to rewrite my equation...

Any positive integer can be factorized as shown below, where $p_0, p_1, p_2, \dots, p_m$ are the prime numbers and $a_0, a_1, a_2, \dots, a_m$ are positive integers.

$$n = p_0^{a_0} \times p_1^{a_1} \times p_2^{a_2} \times \dots \times p_m^{a_m}$$

Then, each divisor of n can all be expressed as shown below.

$$p_0^{b_0} \times p_1^{b_1} \times p_2^{b_2} \times \dots p_m^{b_m}$$

Where $b_0, b_1, b_2, \dots b_m$ can be expressed as the following.

b_0 = any integer in the series 0, 1, 2, 3, ..., a_0

b_1 = any integer in the series 0, 1, 2, 3, ..., a_1

b_2 = any integer in the series 0, 1, 2, 3, ..., a_2

$\vdots$

b_m = any integer in the series 0, 1, 2, 3, ..., a_m

It'd be great if I could just continue to think,

About only the things I love

Whew, that was quite a bit to write. All I had really intended to say was that the product of all the prime numbers each taken to the exponents 0, 1, 2, ... would give the divisor of n.

But now that I've wrote the general formula for each divisor of n, the rest is easy. All I have to do is simply add them.

$$\begin{aligned}\text{Sum of the divisors of } n = {} & 1 + p_0 + p_0^2 + p_0^3 + \cdots + p_0^{a_0} \\ & + 1 + p_1 + p_1^2 + p_1^3 + \cdots + p_1^{a_1} \\ & + 1 + p_2 + p_2^2 + p_2^3 + \cdots + p_2^{a_2} \\ & + \\ & + 1 + p_m + p_m^2 + p_m^3 + \cdots + p_m^{a_m}\end{aligned}$$

Wait. No, that's wrong. That won't give you the sum of all the divisors of n. It would only give you the sum of the elements of $p_0^{b_0}$, $p_1^{b_1}$, and so forth.

As stated before, each divisor of n can be expressed as

$$p_0^{b_0} \times p_1^{b_1} \times p_2^{b_2} \times \cdots \times p_m^{b_m}$$

Wouldn't I need to multiply, not add, the prime number divisor exponentations instead? Agh, writing this out in words is just making it confusing so I'll use numbers and variables instead.

$$\begin{aligned}\text{Sum of the divisors of } n = {} & (1 + p_0 + p_0^2 + p_0^3 + \cdots + p_0^{a_0}) \\ & \times (1 + p_1 + p_1^2 + p_1^3 + \cdots + p_1^{a_1}) \\ & \times (1 + p_2 + p_2^2 + p_2^3 + \cdots + p_2^{a_2}) \\ & \times \cdots \\ & \times (1 + p_m + p_m^2 + p_m^3 + \cdots + p_m^{a_m})\end{aligned}$$

All I'm doing is doing something I enjoy.

My solution to Mukka's homework

Take the positive integer n and factorize it by primes as shown below.

$$n = p_0^{a_0} \times p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_m^{a_m}$$

where $p_0, p_1, p_2, \ldots, p_m$ are prime numbers and $a_0, a_1, a_2, \ldots, a_m$ are positive integers.

Then, the equation for the sum of all the divisors of n is

$$\begin{aligned}\text{Sum of the divisors of } n = {} & (1 + p_0 + p_0^2 + p_0^3 + \cdots + p_0^{a_0}) \\ & \times (1 + p_1 + p_1^2 + p_1^3 + \cdots + p_1^{a_1}) \\ & \times (1 + p_2 + p_2^2 + p_2^3 + \cdots + p_2^{a_2}) \\ & \times \cdots \\ & \times (1 + p_m + p_m^2 + p_m^3 + \cdots + p_m^{a_m})\end{aligned}$$

Umm... Do you talk about...
these math-related problems with...
...that person as well?
That person...?
A-ah- I mean, umm...
T-that uhh girl, you're always seem to be with...
The one who kicked that chair yesterday!!!
.....

She's a very smart girl.
I can't hope to beat her.
She's very knowledgeable on a wide variety of subjects.
Much more so than me
.....
Yes...
Mitka?
Well, the stuff we talk about is usually math-specific,
depending on the sorts of calculations we're doing.

Are the two of you dating?
Ah-

Wha-
What'd I just say!?
And what's with this timing!?
I...
I'm r-r-really sorry about sticking my nose where it doesn't belong!
B-but,
If you two really are in that sort of relationship,
Umm
Calm down
Then it must've been really shameless for me to sit right next to you!
No,
The two of us aren't like that
.....
Um...

Then...
But, if you are going to continue...
Sure thing
Oh, well in that case, I'd really like it if you could continue on tutoring me in math as long as it's not too troublesome...
Milka might... with the chair...
Don't worry, I'll talk to her about it
.....
Oh, good point. She really might kick the chair again.
So you think so too?

"Yeah."
"We're not dating."
Ok!
"I had thought the two of you were dating each other."
"But I guess I was mistaken."

I talked with him, drank tea with him, walked with him, and even went to a bookstore with him!
I'm s-s-superrrr amazing!
What's with me today!?
This is amazing! No, I'm amazing!
What're you, a human-crab?
Not only that,
"We're not dating."
I got him to answer.

My solution to Milka's homework.

Take the positive integer n and factorize it by primes as shown below

$$n = p_0^{a_0} \times p_1^{a_1} \times p_2^{a_2} \times \cdots \times p_m^{a_m}$$

where $p_0, p_1, p_2, \cdots, p_m$ are prime numbers and $a_0, a_1, a_2, \cdots, a_m$ are positive integers.

Then, the equation for the sum of all the divisors of n is

$$\begin{aligned}\text{Sum of the divisors of } n = {} & (1 + p_0 + p_0^2 + p_0^3 + \cdots + p_0^{a_0}) \\ & \times (1 + p_1 + p_1^2 + p_1^3 + \cdots + p_1^{a_1}) \\ & \times (1 + p_2 + p_2^2 + p_2^3 + \cdots + p_2^{a_2}) \\ & \times \cdots \\ & \times (1 + p_m + p_m^2 + p_m^3 + \cdots + p_m^{a_m})\end{aligned}$$

That's the most important piece of news of all!

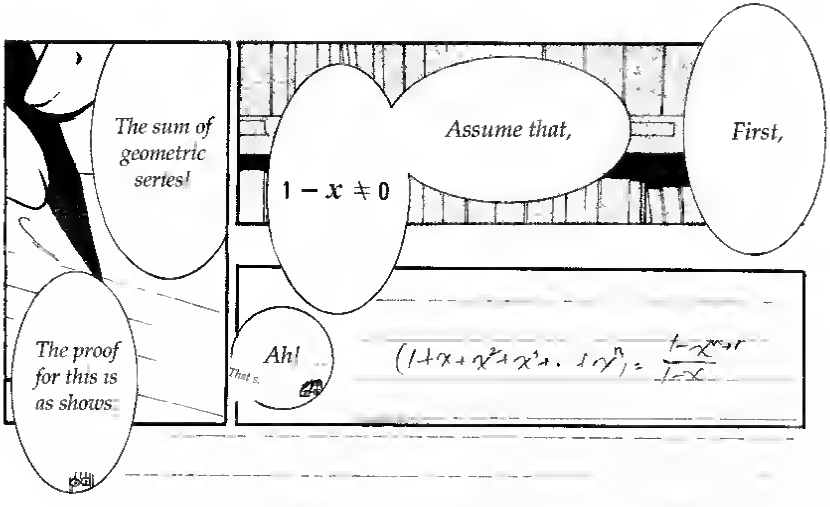
Although it is a little cumbersome

Correct.

There is.

Is there a simpler way to write it?

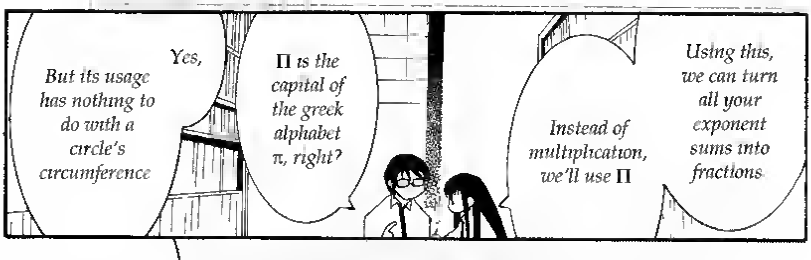
LIBRARY



$1 - x^{n+1} = 1 - x^{n+1}$ factor the left side to get,

$(1-x)(1+x+x^2+x^3+\cdots+x^n) = 1-x^{n+1}$ divide both sides by 1-x to get,

$$1+x+x^2+x^3+\cdots+x^n = \frac{1-x^{n+1}}{1-x}$$


But its usage has nothing to do with a circle's circumference
Yes,
Π is the capital of the greek alphabet π, right?
Instead of multiplication, we'll use Π
Using this, we can turn all your exponent sums into fractions

It's basically just a product version of Σ.

Π→ *P (for product) in the Greek alphabet*
Σ→ *S (for summation) in the Greek alphabet*

$\prod_{k=0}^{n} f(k) = f(0) \times f(1) \times f(2) \times f(3) \times \cdots \times f(n)$

The formal definition for Π is.

Miruka's Solution

So using Π, we can simply write:

Using prime factorization, positive integer n can be written as below

$$n = \prod_{k=0}^{m} p_k^{a_k}$$

where p_k is a prime number and a_k is a positive integer.

Then, the sum of all the divisors of n can be written as the following equation.

$$\text{Sum of the divisors of } n = \prod_{k=0}^{m} \frac{1 - p_k^{a_k+1}}{1 - p_k}$$

It then becomes a lot shorter to write, though we now have to use more alphabets as symbols
I see,
My kouhai.
Who?
Tetra.
Name?
It's about the girl,
that was sitting next to me the other day.
Umm.....
Milka.

So...
I'm actually,
Tutoring her in math
Since she's going to come and ask me more questions in the future...
You don't want me to sit here anymore
Is that it...?

No, of course not.
It's perfectly fine,
For you to continue sitting here.
It's just...

Could
ou just
kick
more
rs?
Got it.
Tutoring at library
Your kouhai
I got it.
Name's Tetra.
Now let's go back to math.

T I P S

[*Chair (Isu in Japanese)*]

A furniture used for sitting. Thus, kicking it is not its intended original use. However in Japanese, its secondary definition means office or position and thus the verb "kick" can be commonly used in conjunction. But only Miika knows just what sort of position (Isu) the chair (Isu) next to me signifies.

数学ガール

Mathematical Girls

Translated by Hox

Morning.

Morning, Milka.
Chapter 4
Here's your umbrella back

It really helped me out yesterday
Oh, that's good
Rotation?
There's no need to memorize that
We're only practicing, after all
Simply look where the unit vector will be displaced to
As usual, Milka doesn't pronounce the word, "vector," like any other Japanese person would

If you like playing around with formulae,
You might find it fun to try rotating the unit vector twice by the angle of θ
Even for θ,
She pronounces it as "theta," like an English-speaking person would
Summer is about to begin.
That is all.
Only the season has changed.
The one sitting next to me,
Is the same old Milka I've always known.

Even I,
LIBRARY
Go to tutor Tetra as usual
Do you know about absolute values?
And yet...
Yes,
I think so
I just take away any negative signs, right?
Pretty much
The absolute value for both 5 and -5 is just 5

Here's the formal definition, however.
The absolute value of x is defined as
$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$
Does this all make sense?
Ah, this reminds me,
Of a question I've had about absolute value signs.
You're technically just taking away the minus sign,
So why would it be -x for half the time?
Well, in mathematical terms, simply saying "taking away the negative sign" is a little vague
But I do understand what you're trying to say
Then
How about changing the minus into a plus?
That's vague as well

|−x|
Consider this problem What would the absolute value of -x be?
For example,
Huh?
For example, what would happen if x= -3?
Well, if x = -3
Not quite.
Well
The answer should be x
$|-x|=x$
Well, since you just take away the minus sign,

|−x| = |−(−3)|
= |3|
= 3
x = −3
−(−3) = 3
|3| = 3
|−x|=−3
Then when x = -3,
If we go with what you answered,
By making |−x|=x,
But of course, the actual answer should be |−x|= 3
Thus,
|−x|=x
|−x|=−x
The correct answer is,
|−x|=−x

Ohh!
I get it now.
So when taking the absolute value of x,
You really have to add a minus sign if x < 0.
Before, I had mistakenly assumed that x would always be a positive number like 3 or 5.
But because we're used to seeing the negative sign when a number is negative, it's easy to mistakenly assume that x will always be a positive number
x is merely a variable
|x|
x = -3
That's the key point you should be aware of
Yes, exactly.

The reason why we use variables such as x for our definitions,
Is to make a definition valid for all cases regardless of the actual number itself.
So defining an absolute value as "taking away the negative," is a little too sloppy
We have to carefully consider all other cases as well
Sometimes, this carefulness may seem even painstakingly strict
But once you become used to this strictness
You'll soon find yourself becoming used to not just formulae but all of math as well

Ah-
It's raining...
Do you not have an umbrella?
I didn't bring one...
The skies were so clear this morning...
Oh.
Well
If you'd like,
You can ..

That reminds me...
Sometimes, yes
A regular one and a collapsible one!
Always carry two umbrellas?
Do you,
.....
No problem
Thanks for lending me an umbrella

Miruka,
Ah Today is
I wonder if she brought her own umbrella today...?
Left for home earlier...
?
Book

Eiei
Hm? Alone?
Weren't you and Milka going to...
We were supposed to but...
She told me she was going to head to the bigger bookstore just up ahead...
But...

It didn't seem like she brought an umbrella today either
And it's pouring quite heavily right now...
If you rotate by θ twice
You first end up with this equation
But then, you should realize that rotating by θ twice is the same as rotating by 2θ

Doing that, you can derive these two equations,
Both of which are trigonometric identities
So what kind of identities are these?
Double angle identities
What, don't you know?
Huh?
They're double angle identities, of course.
...I did know that.

First, start by writing the rotation matrix.

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

When you rotate any vector by an angle of θ, all you're doing is multiplying this rotation matrix to that vector

So, if you wanted to rotate by θ twice, you'd multiply the rotation matrix twice as well

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}^2$$

$$= \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \times \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$= \begin{pmatrix} \cos\theta \times \cos\theta + (-\sin\theta \times \sin\theta) & \cos\theta \times (-\sin\theta) + (-\sin\theta) \times \cos\theta \\ \sin\theta \times \cos\theta + \cos\theta \times \sin\theta & \sin\theta \times (-\sin\theta) + \cos\theta \times \cos\theta \end{pmatrix}$$

Thus,

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}^2 = \begin{pmatrix} \cos^2\theta - \sin^2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{pmatrix}$$

Now, as I've said before, rotating twice by θ is the same as rotating once by 2θ

Replace the left-hand side matrix

$$\begin{pmatrix} \cos 2\theta & -\sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} = \begin{pmatrix} \cos^2\theta - \sin^2\theta & -2\sin\theta\cos\theta \\ 2\sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{pmatrix}$$

Comparing each element in the two matrices, we can derive the below two equations

So basically, we can now express cos2θ and sin2θ in terms of cosθ and sinθ

And so, the matrix on the left hand side of the equation can now be rewritten like so

$$\therefore \cos 2\theta = \cos^2\theta - \sin^2\theta \qquad -\sin 2\theta = -2\sin\theta\cos\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

Or in short, we use θ to express 2θ.

And that's the double angle identity
So in summary, we've derived the identities by using the rotation matrix to turn by different angles.
The key step here was to realize that rotating once by 2θ is the same as rotating twice by θ.
By realizing that the two,
Are in reality, one and the same,
You'll then find....

That wondrous things can happen
Miruka!
Just as I thought
You're gonna catch a cold.
The same old road,
That leads to the bookstore

In any case...!
In...
Ch!
Like a drenched rat...
Achoo!
I'll walk you home
Let's go
But the bookstore's right in front
I don't think,
They'd let in a soaking-wet person
Ahh!

The usual Milka that I know...
And...
I can't let others see her,
When she's completely wet like this.
Abso-lutely not.
You're turning red

The girl standing before me right now...

"You'll then find..."
"Are in reality, one and the same,"
"By realizing that the two,"
"That wondrous things,"
"Will happen."

T I P S

[*Vector*]

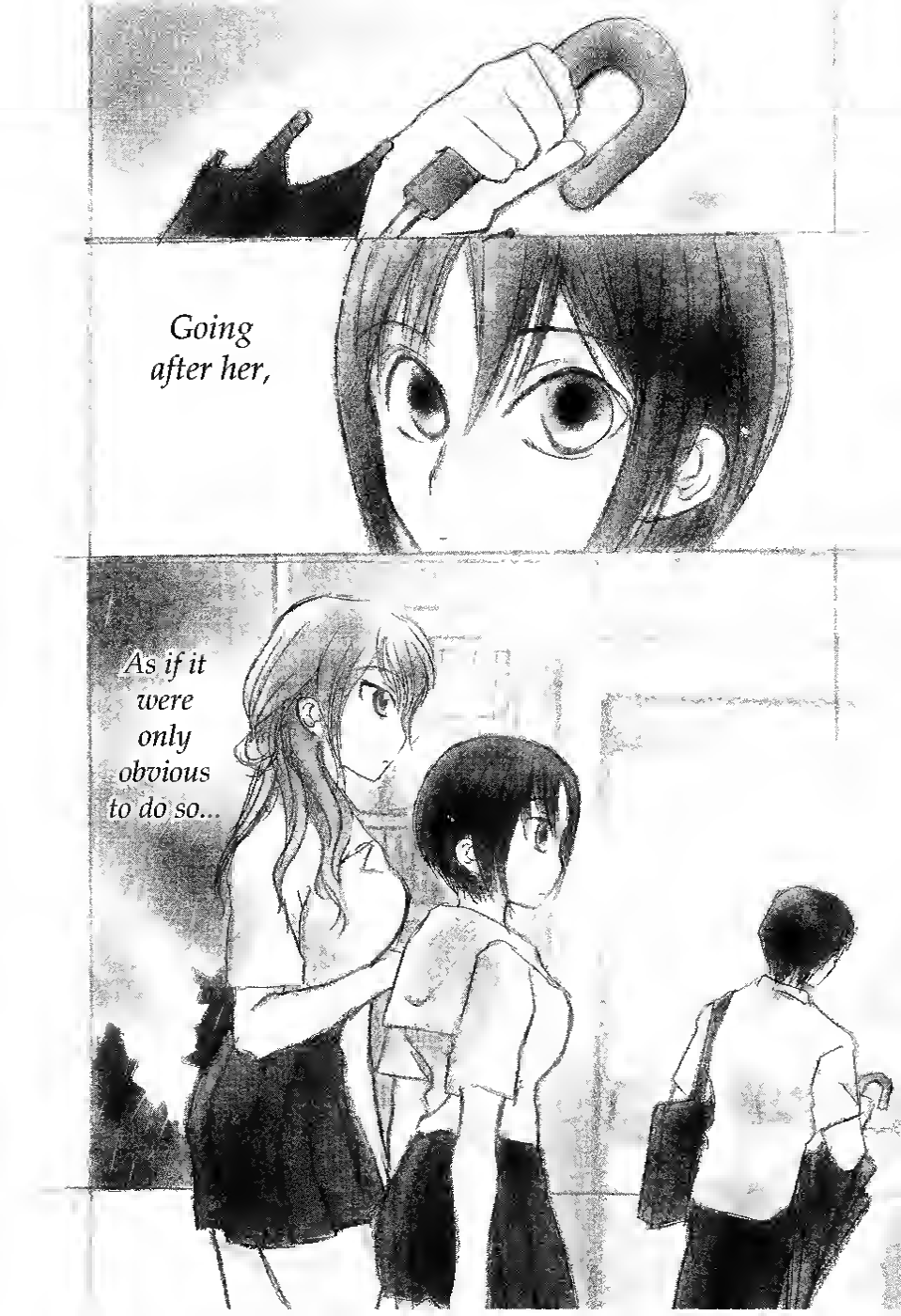
Anything that has bath magnitude and direction. Milka pranaunces it with a "V," while mast Japanese students wauld pranaunce it with a "B." Althaugh cammanly depicted with an arraw sign shawing the direction, that is not always sa. For instance, a paint above a plane can be expressed as a vector where its direction is implied ta be headed away fram the arigin.

数学ガール

Mathematical Girls

Translated by Hox

Chapter 5


Going after her,
As if it were only obvious to do so...

Sigh...
Thank you for lending me your umbrella
It really helped me out during the downpour
yesterday
I want to write so much more,
What is this,
And yet...
Barely two sentences...

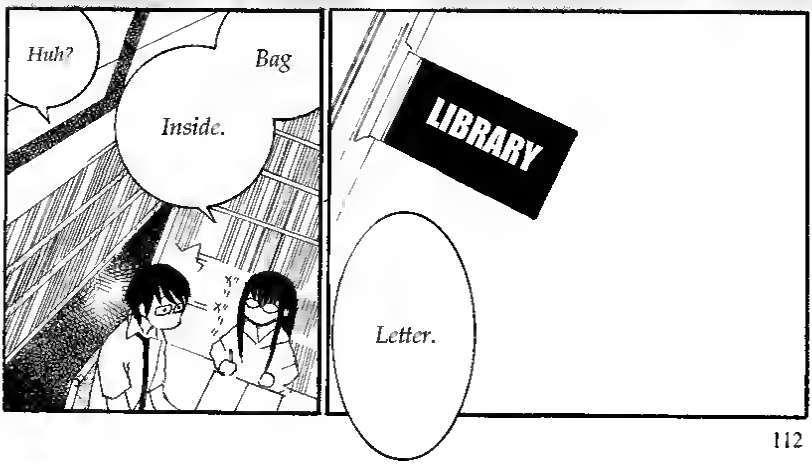
When I first wrote that letter,
Asking if he could teach me in math.
I felt the same way,
No...
This feeling is somehow much stronger than that time.
LIBRARY

"Don't worry, I'll talk to Milka about it."
Ah-
S-sorry...
No no I should be apologizing Are you alright?
But even so,
I just don't think I can...
どしゃ

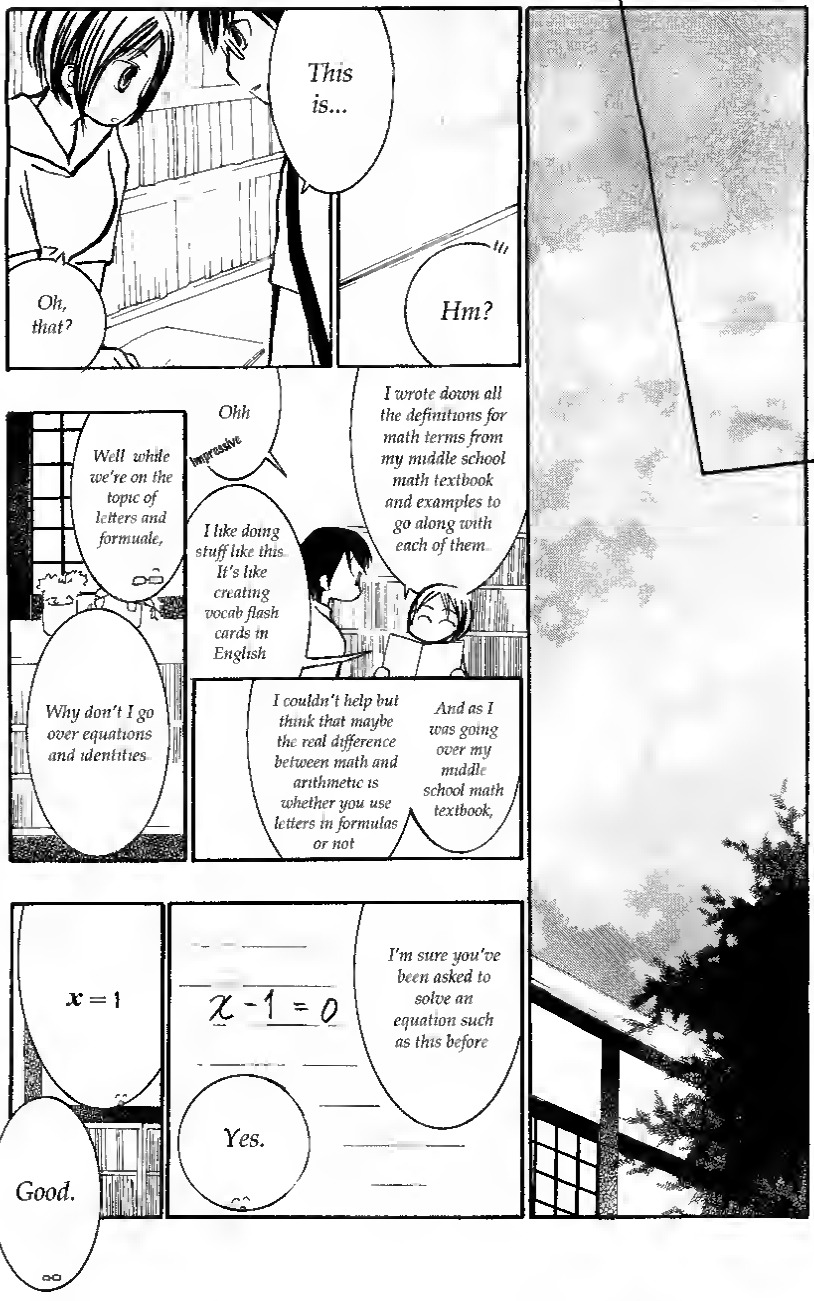
You're...
Current Situation
Me
Mika
Tetra
The two classroom doors
W why her of all people...

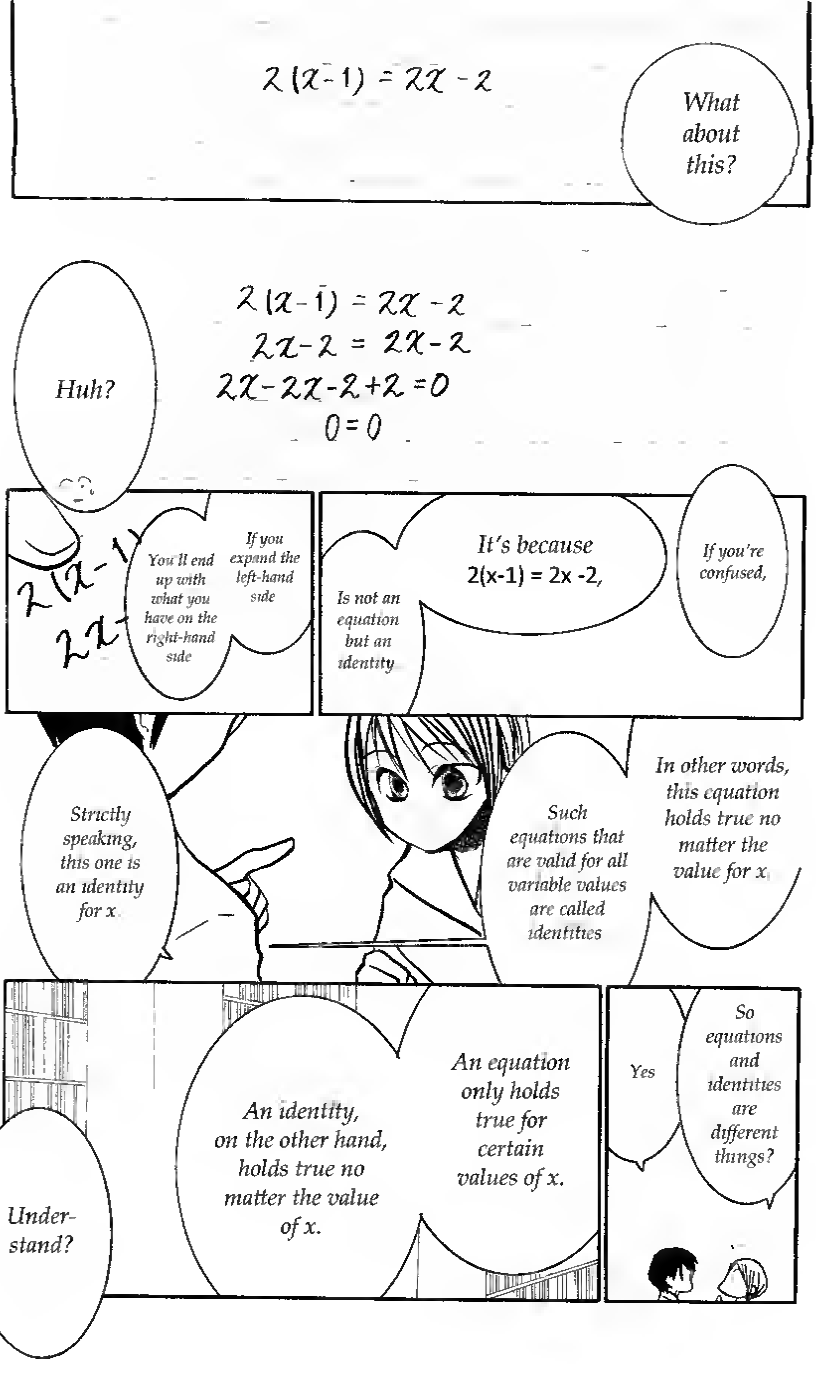
U-umm, thank you very much for yesterday!
Your umbrella's inside the bag.
B-b-bye for now!

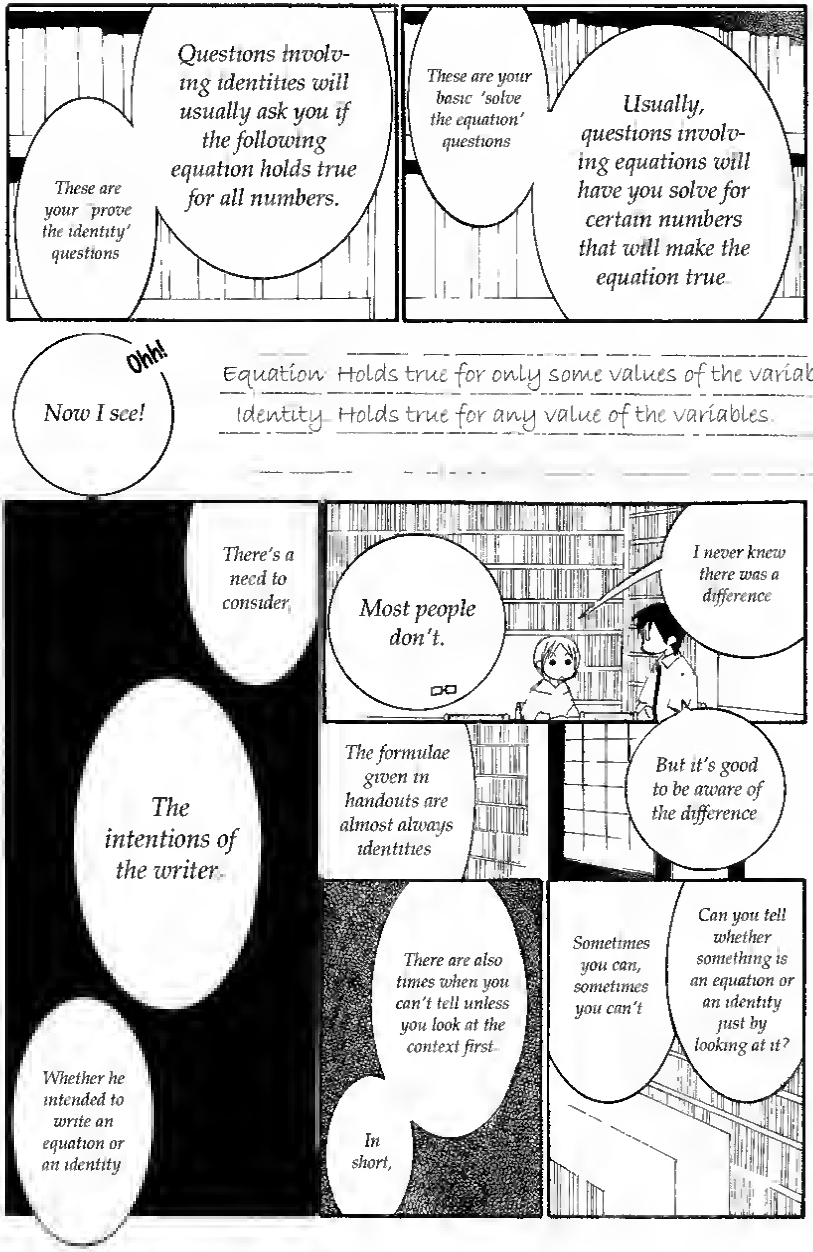
What am I doing...



How did she know...?
Concentrating 100%
She didn't even look at the bag...
Oh...


This is...
Oh, that?
Hm?
I wrote down all the definitions for math terms from my middle school math textbook and examples to go along with each of them.
Ohh
Impressive
I like doing stuff like this. It's like creating vocab flash cards in English
Well, while we're on the topic of letters and formulae,
Why don't I go over equations and identities
And as I was going over my middle school math textbook,
I couldn't help but think that maybe the real difference between math and arithmetic is whether you use letters in formulas or not
I'm sure you've been asked to solve an equation such as this before
x - 1 = 0
Yes.
x = 1
Good.


2(x-1) = 2x - 2
What about this?
Huh?
2(x-1) = 2x - 2
2x-2 = 2x-2
2x-2x-2+2 = 0
0 = 0
2(x-1)
2x-
You'll end up with what you have on the right-hand side
If you expand the left-hand side
Is not an equation but an identity
It's because 2(x-1) = 2x -2,
If you're confused,
Strictly speaking, this one is an identity for x.
Such equations that are valid for all variable values are called identities
In other words, this equation holds true no matter the value for x.
Understand?
An identity, on the other hand, holds true no matter the value of x.
An equation only holds true for certain values of x.
Yes
So equations and identities are different things?


These are your 'prove the identity' questions
Questions involving identities will usually ask you if the following equation holds true for all numbers.
These are your basic 'solve the equation' questions
Usually, questions involving equations will have you solve for certain numbers that will make the equation true.
Ohh!
Now I see!
Equation Holds true for only some values of the variables
Identity Holds true for any value of the variables
There's a need to consider
The intentions of the writer.
Whether he intended to write an equation or an identity
Most people don't.
I never knew there was a difference
The formulae given in handouts are almost always identities
But it's good to be aware of the difference
There are also times when you can't tell unless you look at the context first.
In short,
Sometimes you can, sometimes you can't
Can you tell whether something is an equation or an identity just by looking at it?

The writer's intentions...?
For instance, identities are used when showing alternate forms.

$$
\begin{aligned}
(x+1)(x-1) &= (x+1)\cdot x - (x+1)\cdot 1 \\
&= x\cdot x + 1\cdot x - (x+1)\cdot 1 \\
&= x\cdot x + 1\cdot x - x\cdot 1 - 1\cdot 1 \\
&= x^2 + x - x - 1 \\
&= x^2 - 1
\end{aligned}
$$

Take a look at this example

In other words, it's a chain of identities

This emphasizes the point that it holds true no matter the value of x

The equals sign continues all the way down

Thus, this last line is also an identity

Oh, ok

$$(x+1)(x-1) = x^2 - 1$$

Now ..

So make sure to go through it line by line instead of thoughtlessly arriving at the last line

"S-so many equations..."

The whole purpose behind this long chain of identities is to show in slow motion how some of these identities are derived.

$x^2-5x+6 = (x-2)(x-3)$
$= 0$

What about this example?

Like so.

$x^2-5x+6 = (x-2)(x-3)$

The above line is true for all x.

Which means,

The first line is an identity,

This is what it means

Instead of trying to solve: $x^2-5x+6=0$

We replace the left-hand side with the previously noted identity,

to instead solve: $(x-2)(x-3)=0$

And so,

The second line, however, is an equation

Unlike equations and identities, there's no need to solve for nor prove these definition lines

It's just decided for convenience

Other than equations and identities, there are sometimes also definition lines

So that's how you're supposed to read it...

Provided that...

We could replace it with the simpler s,

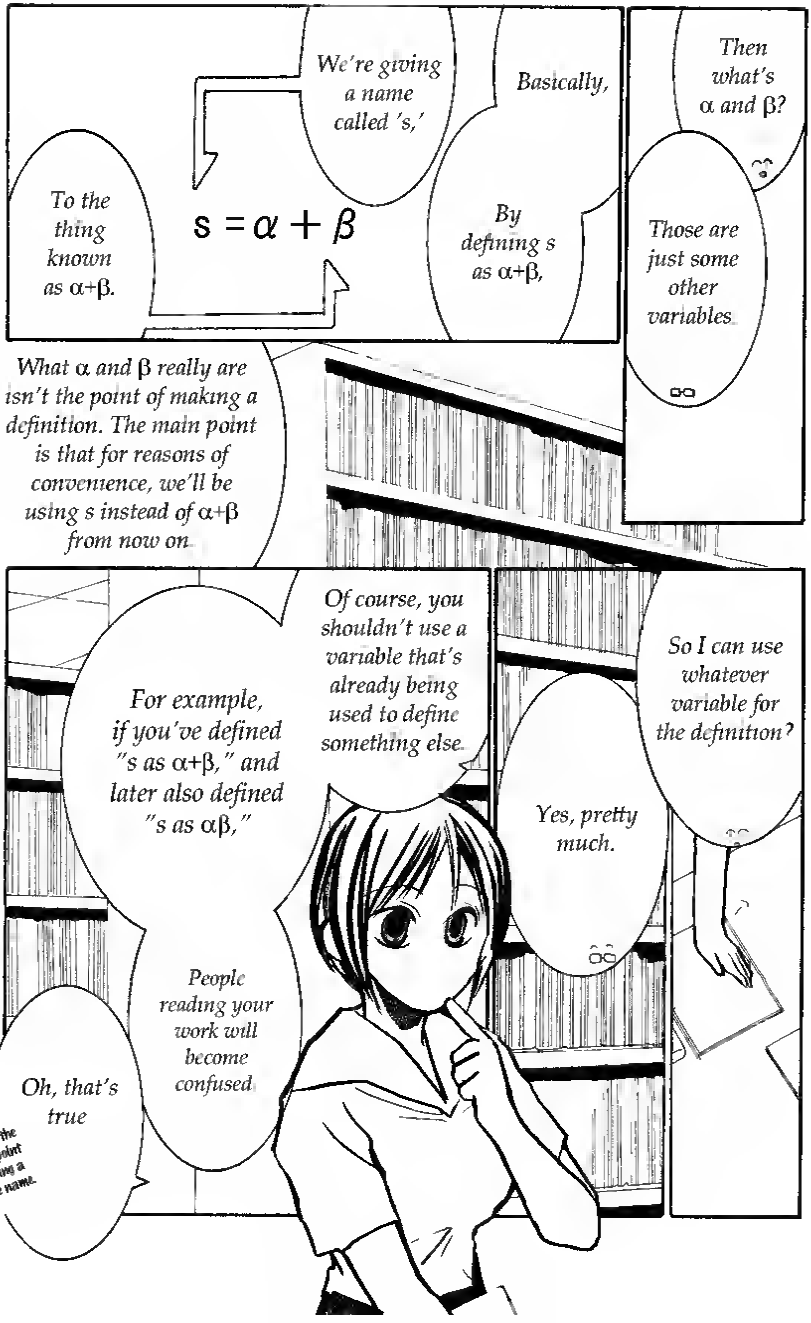
For example, instead of writing $\alpha+\beta$,

That you decide to use to replace something that would otherwise be more complicated

Basically, you have a defn tion,

Decided for convenience?

S = α + β (defintion line example)
We define our s,
Like so.
It doesn't have to be s. It could be t, u, or whatever.
W-why is it s...?
I have a question!
はいはいはいは


We're giving a name called 's,'
Basically,
Then what's α and β?
To the thing known as α+β.
s = α + β
By defining s as α+β,
Those are just some other variables.
What α and β really are isn't the point of making a definition. The main point is that for reasons of convenience, we'll be using s instead of α+β from now on.
Of course, you shouldn't use a variable that's already being used to define something else.
So I can use whatever variable for the definition?
For example, if you've defined "s as α+β," and later also defined "s as αβ,"
Yes, pretty much.
People reading your work will become confused.
Oh, that's true
It'd lose the whole point of giving a unique name.

"Ah, this must be a definition"
But in any case, don't panic if you see a new letter all of a sudden
In fact, thinking like so will be more helpful
So to pick different names instead of going with the common convention is not recommended
While i is used for the imaginary number,
Also, π is generally used as the ratio of a circle's circumference to its diameter,
Because I'm sure,
Oh
That's how you'll know for sure
We shall hereby define s as the following
The definition will be right there on the page
$s = \alpha+\beta$
Ok, I'll try
Or perhaps something else?
And seeing if they're equations, identities, or definitions?
Ah, why don't you try looking through the equations written in your textbook,

The thmgs you tell me in these math lessons,
Are always really helpful.
It's just...
The math that you usually do...
Is umm...
So much more advanced than what you teach me.
So...
I was wondering if you might not find teaching me bother-some....

A little too late to ask about that now, don't you think?
It is it!?
Well...
Since you're so kind...
That reminds me,
You were like this from the start too, weren't you, Tetra?
W-what do you mean!?
I'm talking about the letter in which you asked me to teach you in math
O-oh...

In math,
There's an enormous amount of formulae.
But each and every one of them was written by someone in order to tell us something.
Behind every equation, identity, and definition,
There's a writer who wants to send us all a message.
Their thoughts and feelings are conveyed by those lines.
I look and study these formulae day after day.
Because I want to understand and communicate with their feelings.

Since you've also started developing an interest in math
I'd be very happy.
If you could come to understand their feelings as well
Back when,
I wrote my first letter to him...

I think,

That I really wanted to say something else than what I wrote.

T I P S

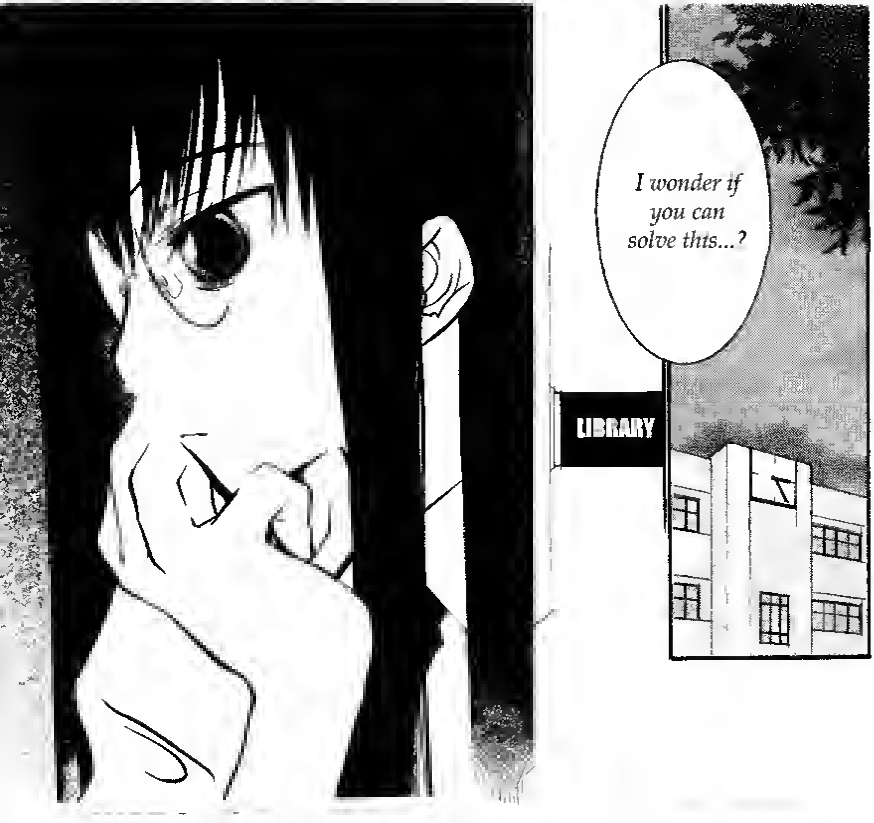
[*Identity*]

Equations which hold true for all values of variables such as "x=x" or "x+x=2x." It does not necessarily need to have variables. For instance, "1+2=3." Most of the formulae you study in math are identities. For example, "$(x+y)(x-y)=x^2-y^2$" is an identity.

数学ガール

Mathematical Girls

Translated by Hox

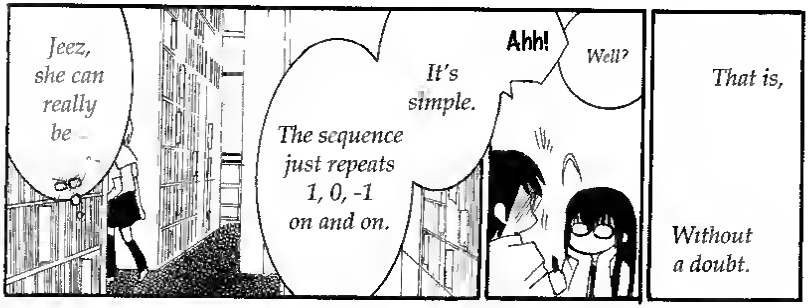


Find a general formula for a_n by using n.

n	0	1	2	3	4	5	6	7
a_n	1	0	−1	0	1	0	−1	0

I'm sure nobody talks with her more than I do.

Chapter 6


Jeez, she can really be
It's simple.
The sequence just repeats 1, 0, -1 on and on.
Ahh!
Well?
That is,
Without a doubt.

Am I wrong?
So that's what you've assumed...
It's sort of like an oscillation.
Well... Your line of thinking isn't entirely wrong...
"We're not dating."
"I had thought the two of you were dating each other, but I guess I was just mistaken."
That much is true...
An oscillation, huh

That we're not actually dating.
But...
It's true,
Try writing down that oscillation pattern into a formula then.
LIBRARY
Senpai!
Are you going to be staying here until after-school?
Yeah,
I was thinking of staying here calculating stuff until Ms. Mizutani tells me I'll have to leave.

For example, consider this example
Form...?
When reading formulae, it's important to pay attention to the form of the expression
This particular example is an equation
$(x-\alpha)(x-\beta)=0$
Yes, exactly
Factorization simply means breaking down a product into its factors. So in prime factorization, you break down a number into its prime number factors or divisors
Oh, is the word "factor" what people refer to when they talk about "factorization?"
The individual elements that are multiplied is what's known as a factor or a divisor
Or to be more specific, this is a product since two things are being multiplied
$(x-\alpha)(x-\beta)=0$
Factor Factor
$(x-\alpha)\times(x-\beta)=0$ × refers to multiplication
$(x-\alpha)\cdot(x-\beta)=0$ · refers to multiplication
$(x-\alpha)(x-\beta)=0$ multiplication sign is not written and implied,
The multiplication sign × is often written in 2 other notations so keep in mind that all 3 forms shown here mean the same thing. In the case here, they're all product equations
Yup, I got it
Because the entire product equals zero, one has to equal 0
That is precisely because this equation is a product
By the way, in the example here, at least one of the factors must be equal to zero.

So saying "at least" is another strict way to express things precisely, right?
Yes
Ohh
Well instead of saying "one is zero" you should technically say, "At least one factor should be 0"
After all, both factors could be 0.
So,
Even in math,
The words are crucial, too!

"No."

"Are the two of you dating?"

Even so...

Things I've put into words,
And...
Things I haven't put into words...
Right.
Since at least one of the factors must equal 0, that means x-α=0 or x-β=0. Simplying further, x=α, β.
So as I was saying...
Senpai?
Now, is this an equation?
$(x-\alpha)(x-\beta)=x^2-\alpha x-\beta x+\alpha\beta$
Let's look at this example
Next,
It refers to the individual components within the expression
Terms?
Looking closely, we see that the product of 2 factors in the left-hand side has yielded 4 terms in the right-hand side
Good
No, it's an identity
Here, I'll put in some parentheses and labels to make it easy to understand
Expand
$(x-\alpha)(x-\beta) = (x^2)+(-\alpha x)+(-\beta x)+(\alpha\beta)$
Factor Factor Term Term Term Term
Factorize

$x^2-\alpha x-\beta x+\alpha\beta$
Yes
Do you know what we should do?
By the way, this expression has yet to be fully simplified
Ok
We have to combine the first-order terms of x
Oh, by the way, we call terms like ax and bx which only have x to the first power, "first-order terms."
Terms, not things
We should group all the things that have just x and
I never knew it was called that.
What you just did is a process known as "grouping like terms"
Oh
Like so.
$x^2+(-\alpha-\beta)x+\alpha\beta$
Grouped the terms
Correct.
Normally, we like to put the negative sign out in front, however
Now, time for a little quiz
$x^2-(\alpha+\beta)x+\alpha\beta$

$(x-\alpha)(x-\beta) = x^2 - (\alpha+\beta)x + \alpha\beta$

Is this an equation or an identity?

Correct!

An identity

Oh, this is what we just got after expanding and grouping the like terms together. So it should be...

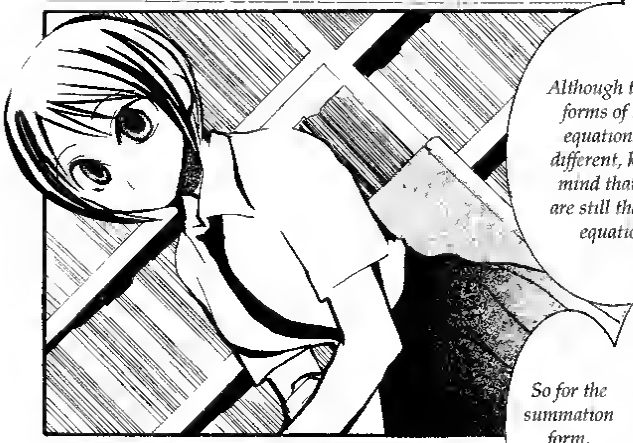
$(x-\alpha)(x-\beta) = 0$ Product Form

This equation that I showed you first is in the form of a product

Continuing on...

$x^2 - (\alpha+\beta)x + \alpha\beta = 0$ Summation Form

But using the identity that we derived, we can switch the equation into the form of a sum instead



Although the two forms of these equations are different, keep in mind that they are still the same equation

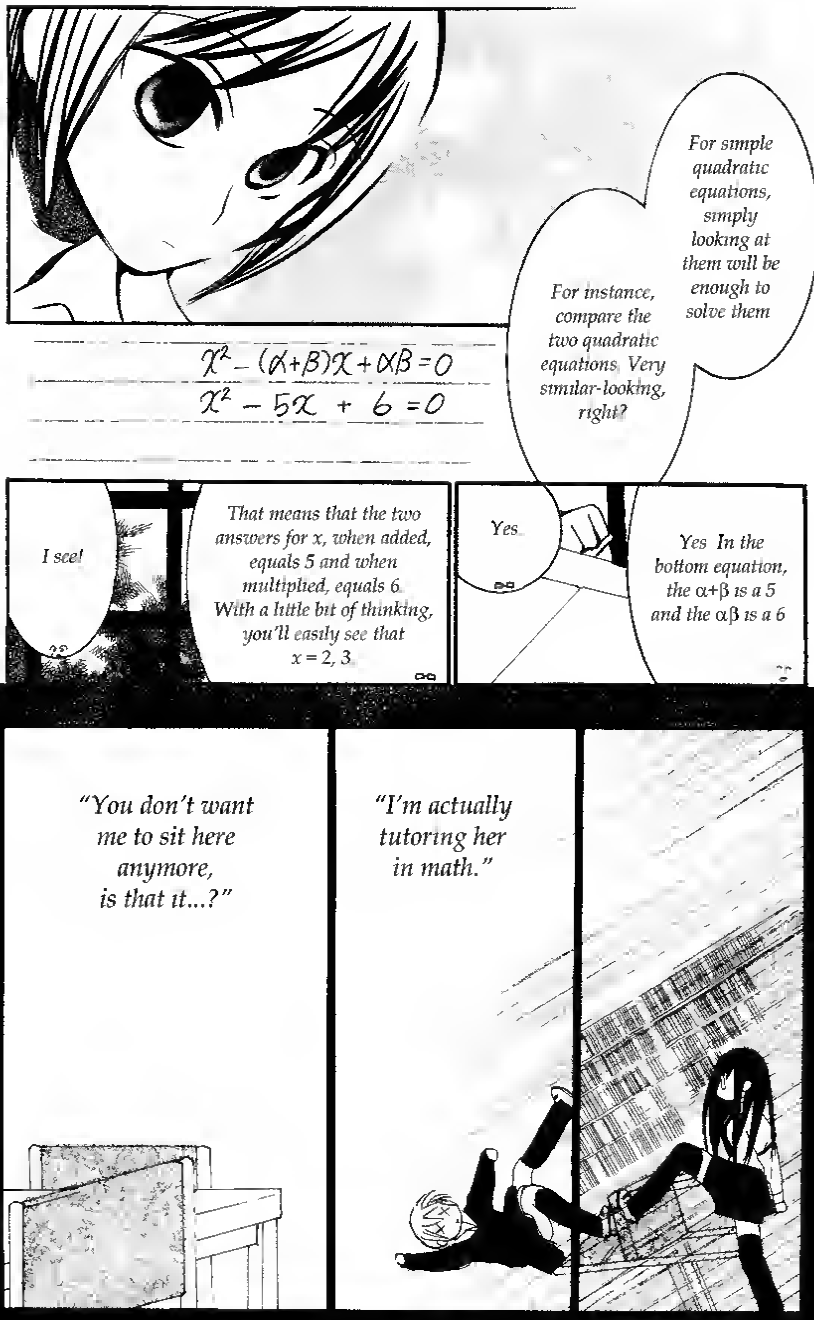
So for the summation form, $x=\alpha, \beta$

And since they're still the same equation, the answers we solved for x will be the same.

$(x-\alpha)(x-\beta)=0$ Product Form ($x=\alpha, \beta$)

$\Updownarrow$

$x^2-(\alpha+\beta)x+\alpha\beta=0$ Summation Form ($x=\alpha, \beta$)


For simple quadratic equations, simply looking at them will be enough to solve them
For instance, compare the two quadratic equations. Very similar-looking, right?
$x^2-(\alpha+\beta)x+\alpha\beta=0$
$x^2-5x+6=0$
I see!
That means that the two answers for x, when added, equals 5 and when multiplied, equals 6. With a little bit of thinking, you'll easily see that $x=2, 3$.
Yes.
Yes. In the bottom equation, the $\alpha+\beta$ is a 5 and the $\alpha\beta$ is a 6
"You don't want me to sit here anymore, is that it...?"
"I'm actually tutoring her in math."

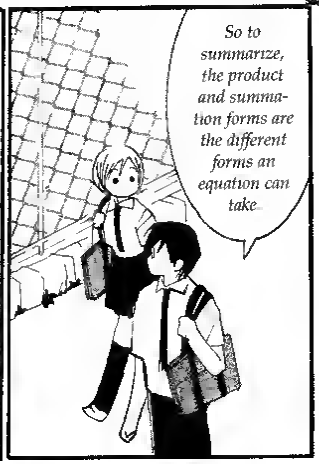
"Let's go back to math."

"No, of course not."

Far from being precise in meaning,

But rather vague and ambiguous.

So when asked to solve an equation, the deeper meaning might be to ask you to make a product form!
Ah!
Answers that might not be obvious in the summation form might be as clear as day when looking at the product form

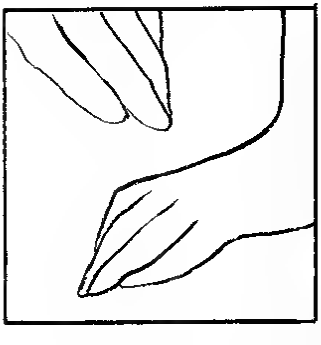


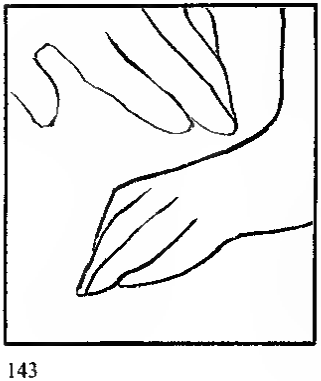
Can't understand...

With only vague words,

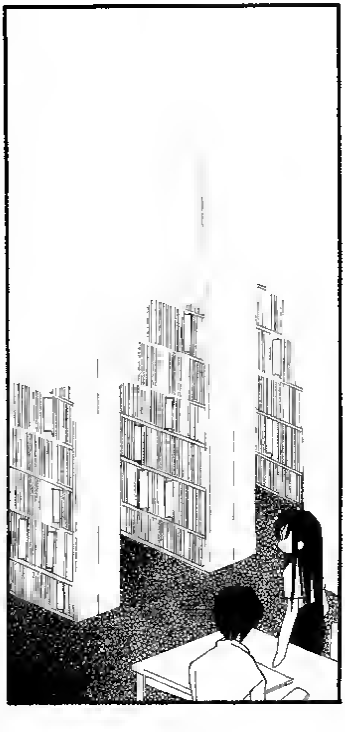
Simply leaving things off,

Won't make anybody,





Understand



Alright, so let's see your general formula for this oscillation then.
$a_n = \begin{cases} 1 & (n = 0, 4, 8, \ldots, 4k, \ldots) \\ 0 & (n = 1, 3, 5, 7, \ldots, 2k+1, \ldots) \\ -1 & (n = 2, 6, 10, \ldots, 4k+2, \ldots) \end{cases}$
All I really had to do was split it off into cases.
A-
?
No food, huh.
Though I personally wouldn't see it as an oscillation
Hmm. Yes, that's correct,

Find a general formula for a_n using n
n | 0 1 2 3 4 5 6 7
b_n | 1 i -1 -i 1 i -1 -i
What will the general formula be?
Try thinking about this problem.
In this sequence, when n is even, b_n is 1 or -1, while if n is odd, b_n is i or -i.
What other i would I be talking about if not the imaginary number?
By i, you mean $\sqrt{-1}$?
Assuming that you're taking this sequence to mean only an oscillation
Another oscillating pattern, huh
.....
That's not wrong either
Why, are there other things that we can interpret this sequence as?

Picture a complex plane

Complex Number ⟷ Coordinate

$x+yi \longleftrightarrow (x, y)$

A coordinate plane in which the x-axis represents real numbers while the y-axis represents the imaginary numbers

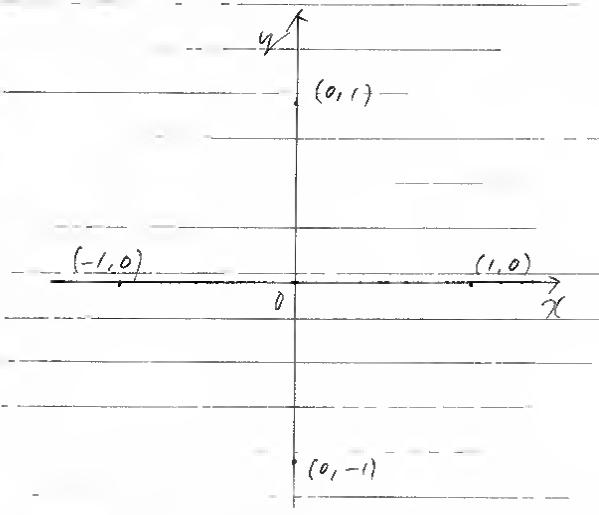
Now think of the series b_n as a sequence of complex numbers

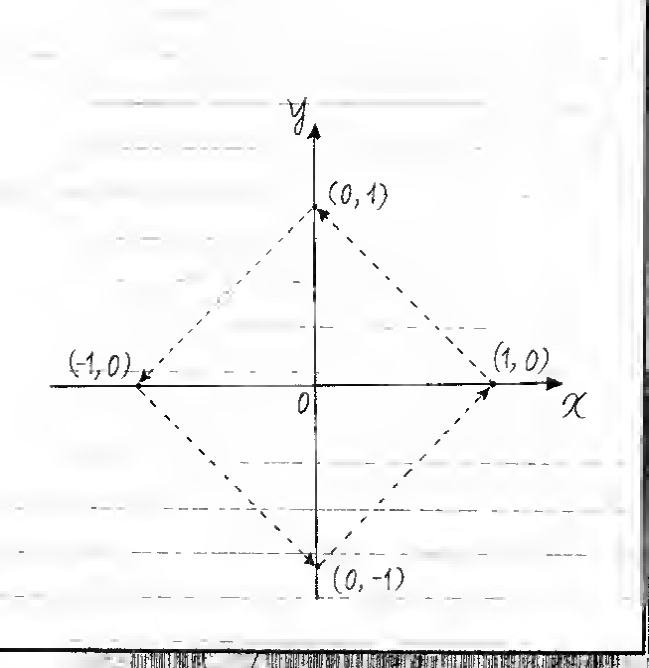
1 is now 1+0i, i is now 0+i, and so on and so on...

$1+0i,\ 0+i,\ -1+0i,\ 0-i,\ 1+0i,\ 0+i,\ -1+0i,\ 0-i,$

$(1,0), (0,1), (-1,0), (0,-1), (1,0), (0,1), (-1,0), (0,-1),$

Then convert these new complex numbers into points on the complex plane.





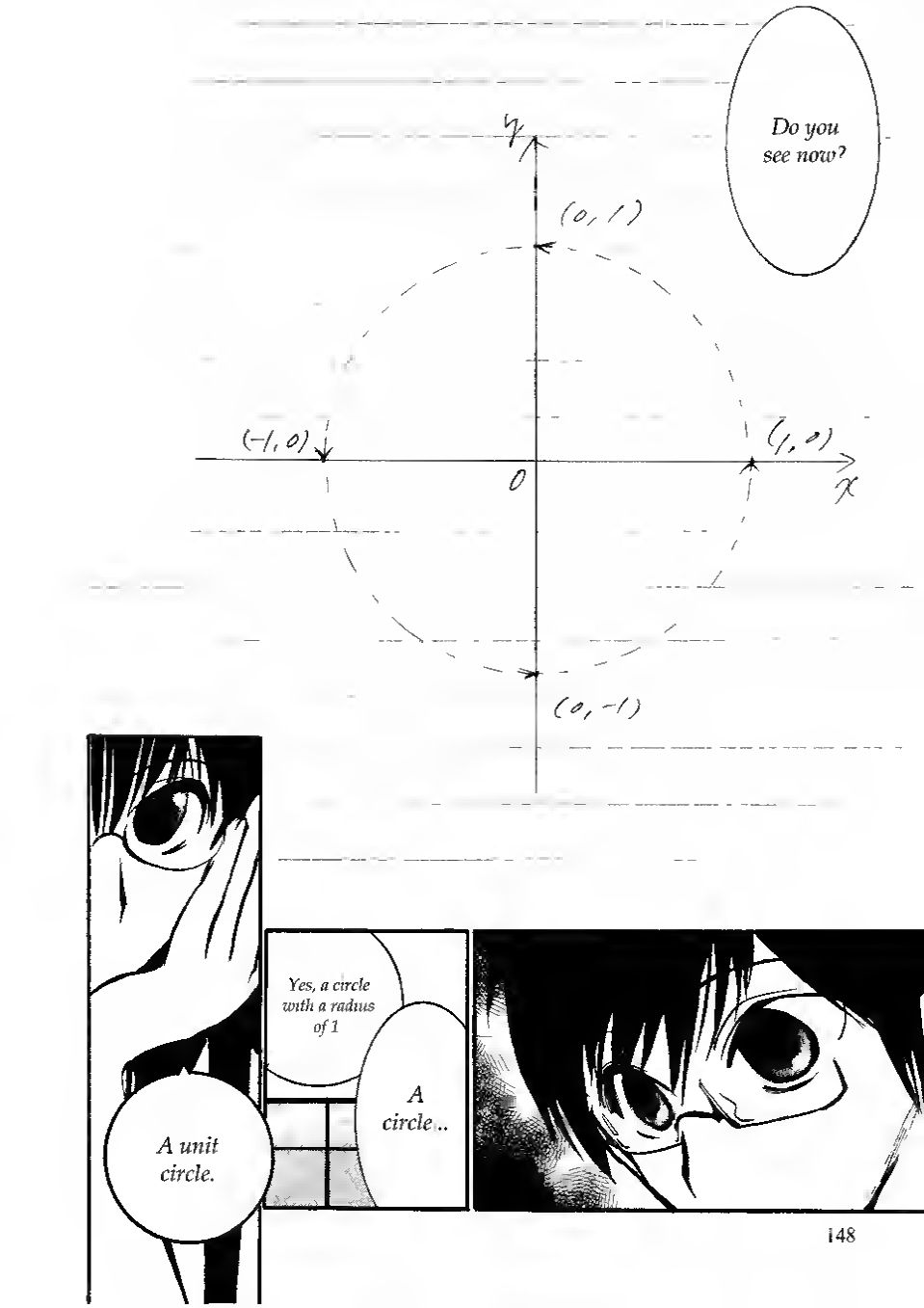
So it'll then traverse in the shape of a diamond or a square.

Not quite.

That's just your assumption, though not entirely wrong

You mean it can be something other than a quadrilateral?

You're rather dense, aren't you?

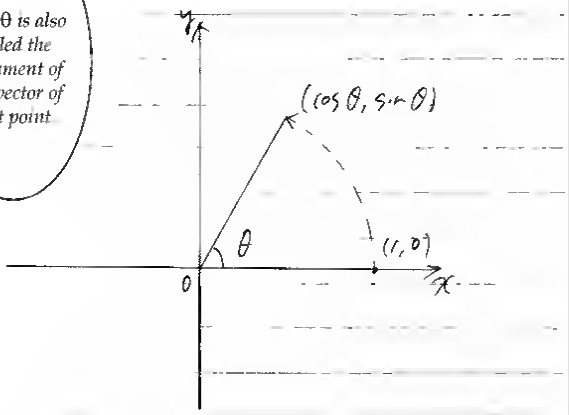

Do you see now?
y
(0, 1)
(-1, 0)
0
(1, 0)
x
(0, -1)
A unit circle.
Yes, a circle with a radius of 1
A circle..

A unit circle...

That's one form the sequence could trace out

A unit circle centered at the origin of a complex plane

Yes, θ is also called the argument of the vector of that point



Any point on this unit circle can be represented as cosθ+isinθ.

$\cos\theta + i\sin\theta$

Right...

And θ is the angle made between the vector (1,0) and the point

Complex Number ⟷ Coordinate

$\cos\theta + i\sin\theta \leftrightarrow (\cos\theta, \sin\theta)$

This is the relationship between a complex number and its coordinate

So, first... How can we write that as complex numbers?

Now going back to the series b_n...

Instead of assuming a square, let's assume a circle that's been devided into 4 equal parts

$\cos 0 \cdot \frac{\pi}{2} + i \sin 0 \cdot \frac{\pi}{2}$

$\cos 1 \cdot \frac{\pi}{2} + i \sin 1 \cdot \frac{\pi}{2}$

$\cos 2 \cdot \frac{\pi}{2} + i \sin 2 \cdot \frac{\pi}{2}$

$\cos 3 \cdot \frac{\pi}{2} + i \sin 3 \cdot \frac{\pi}{2}$

Writing them out, we see that the 4 complex numbers are as shown here.

Since we're dividing the circle into 4 equal parts, we're dividing θ by 90°, or rather, π/2 radians. And so, θ = 0, π/2, π, 3π/2,

$b_n = \cos n \cdot \frac{\pi}{2} + i \sin n \cdot \frac{\pi}{2} \quad (n = 0, 1, 2, 3, \cdots)$

This allows us to make this general formula for b_n

Now let's go back to the first question I gave you regarding a_n

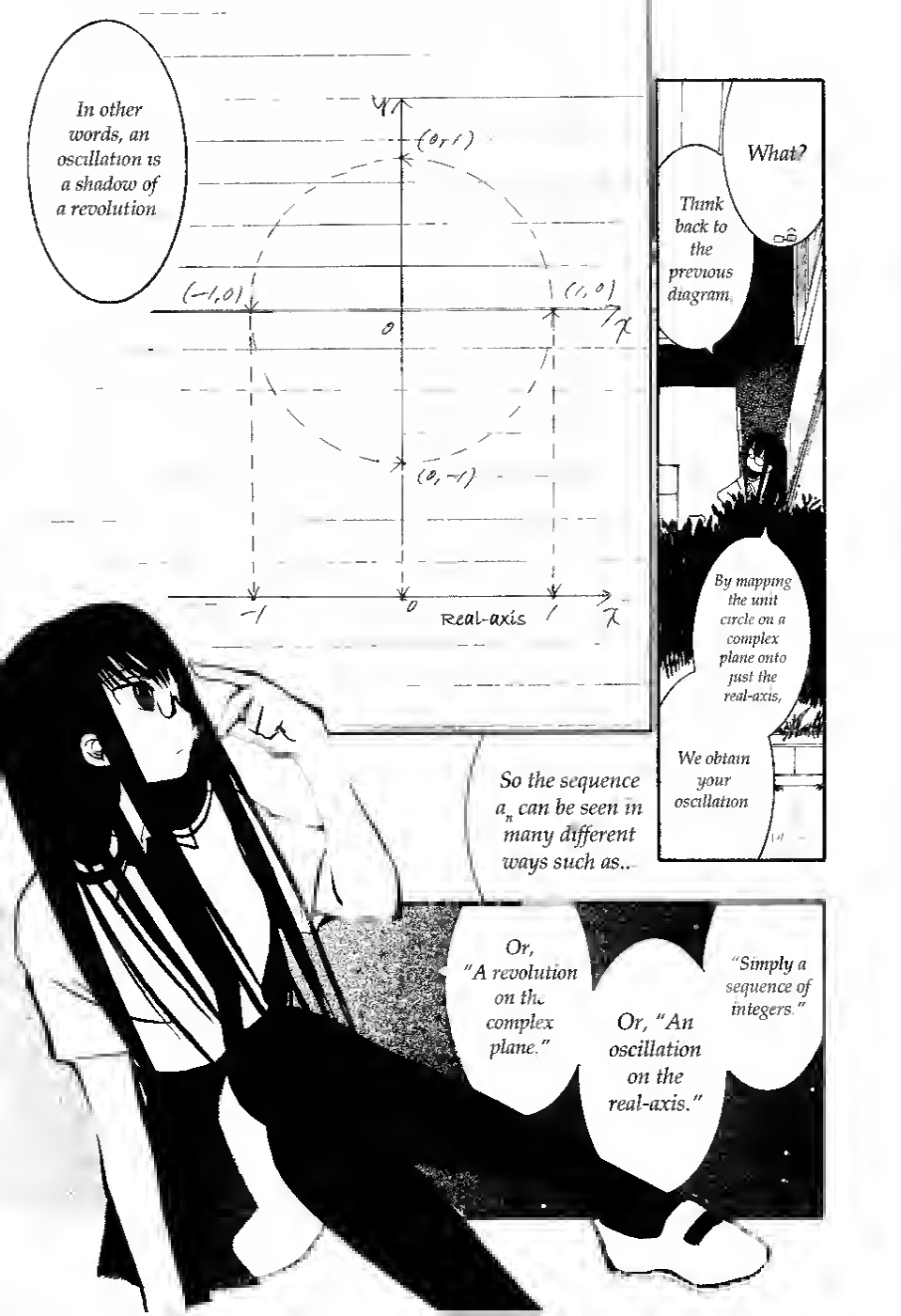
$\langle a_n \rangle = \langle 1, 0, -1, 0, 1, 0, -1, 0, \cdots \rangle$

You told me that a_n was simply oscillating between 1, 0, and -1.

That squence, too, can be rewritten like so.

$a_n = \cos n \cdot \frac{\pi}{2} \quad (n = 0, 1, 2, 3, \cdots)$

……


In other words, an oscillation is a shadow of a revolution
(0,1)
(-1,0)
(1,0)
0
(0,-1)
-1
0
1
Real-axis
What?
Think back to the previous diagram.
By mapping the unit circle on a complex plane onto just the real-axis,
We obtain your oscillation
So the sequence a_n can be seen in many different ways such as..
Or, "A revolution on the complex plane."
Or, "An oscillation on the real-axis."
"Simply a sequence of integers."

You can think about the possibilities in higher-dimensions.
But once you yourself realize that your line of thinking is limited to the first-dimension,
Of course, to actually find these forms hidden amidst the shadows is difficult
So as you can see, alternate forms can be hidden in higher dimensions
The key point is the ey...
Finding out what numbers come next given a part of a sequence is not a simple quiz
Not these eyes, however...
It's a task asking you to consider the possibility of hidden forms

But rather the mind's eye.

What I've been looking at,
Has only been limited to the first-dimension.
If so, then I...

T I P S

[*Like Terms*]

The terms with the same variables and pawers in an expressian.
Far example, 2x and 3x are like terms since they bath contain x^1.
Thus, 2x+3x can then be simplified as (2+3)x which equals 5x
On the ather hand, 2x and 3y are not like terms since they cantain different variables

数学ガール

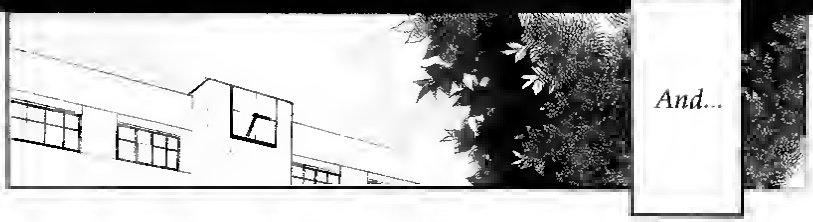
Mathematical Girls

Translated by Hox

I wanted to know,
Milka's feelings,
So I...

But in the end,

The matter ended still trapped in a haze.



And...

Chapter 7

The next day,
Miruka was still the same as ever
No-

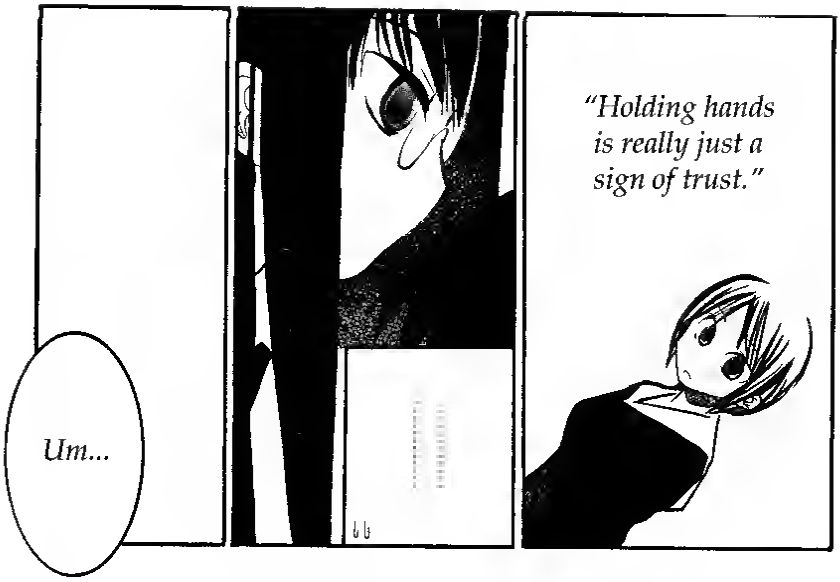
The new piece is done
So let's go play the piano!

In fact, she's become even harder to read.
Girls tend to hold hands with each other a lot, don't they?
Ah, that's true.

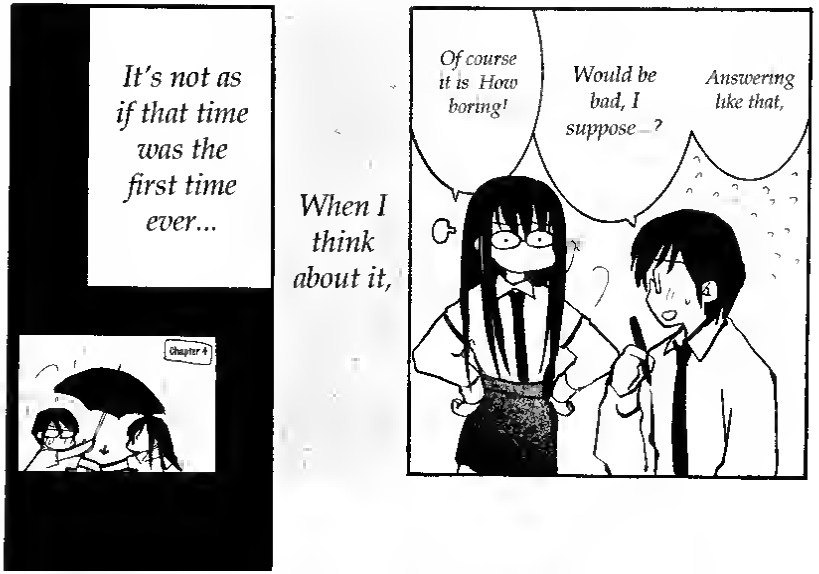
R-right...
Or if they do, there's a special meaning behind that...
Right.
But it doesn't seem like that's the case for guys.
But you know senpai
I think that the act of holding hands is really just a sign of trust.
Even if it's someone of the opposite gender...?

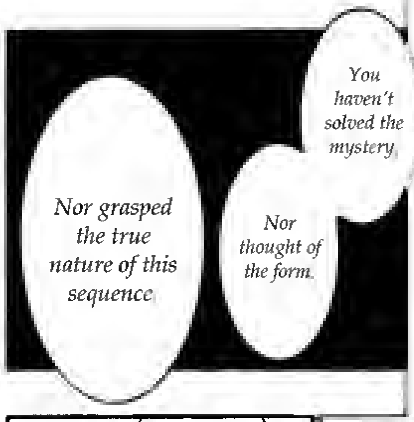
Tetra?
Very friendly isn't it...
Where did you- !?
Pet me!
S-sorry about that.
Umm...

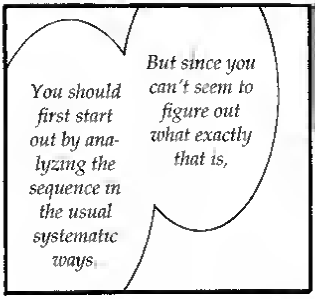
n	0	1	2	3	4	5
C_n	1	$\frac{-1+\sqrt{3}i}{2}$	$\frac{-1-\sqrt{3}i}{2}$	1	$\frac{-1+\sqrt{3}i}{2}$	$\frac{-1-\sqrt{3}i}{2}$

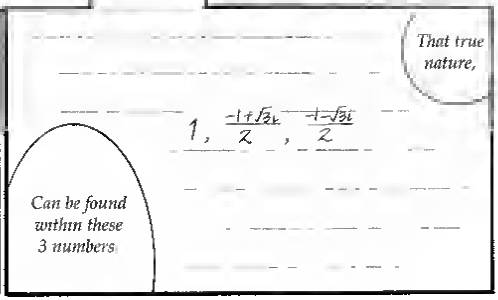


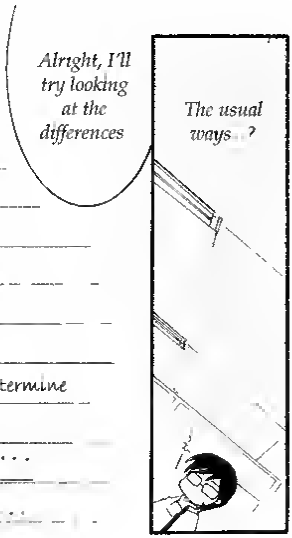
$(1, \frac{-1+\sqrt{3}\,i}{2}, \frac{-1-\sqrt{3}\,i}{2}$, these 3 numbers repeat so on and so on)


It's not as if that time was the first time ever...
Chapter 4
When I think about it,
Of course it is How boring!
Would be bad, I suppose...?
Answering like that,




You should first start out by analyzing the sequence in the usual systematic ways
But since you can't seem to figure out what exactly that is,





For every $\langle c_n \rangle$, we shall define $\langle d_n \rangle$ as follows

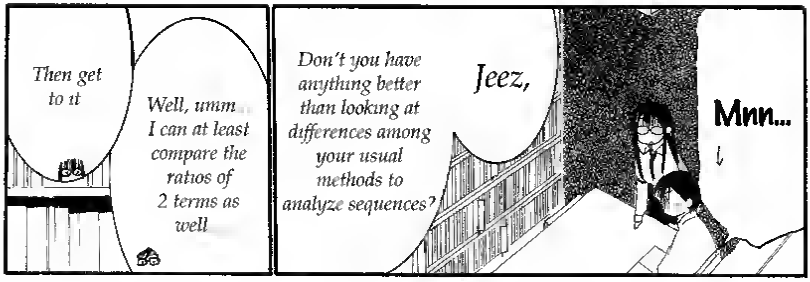
$$d_n = c_{n+1} - c_n \quad (n = 0, 1, 2, \cdots)$$

c_0 c_1 c_2 c_3 c_4 c_5 ...

d_0 d_1 d_2 d_3 d_4

$c_1 - c_0$, $c_2 - c_1$, $c_3 - c_2$, ... will be calculated to determine the values of $\langle d_n \rangle$

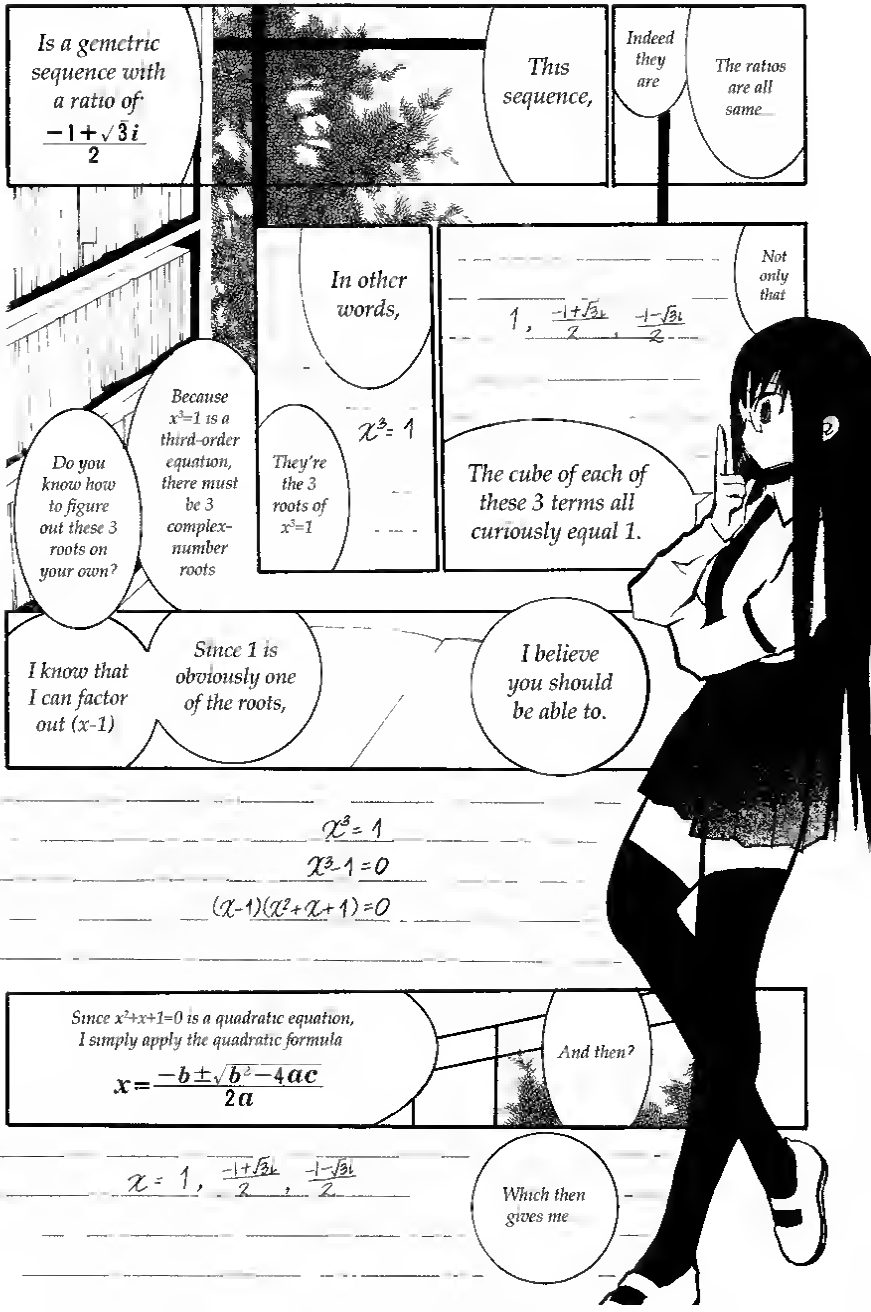
n	0	1	2	3	4	5	...
d_n	$\frac{-3+\sqrt{3}i}{2}$	$-\sqrt{3}i$	$\frac{3+\sqrt{3}i}{2}$	$\frac{-3+\sqrt{3}i}{2}$	$-\sqrt{3}i$	$\frac{3+\sqrt{3}i}{2}$	...


Then get to it
Well, umm... I can at least compare the ratios of 2 terms as well
Don't you have anything better than looking at differences among your usual methods to analyze sequences?
Jeez,
Mnn...

Let us define sequence e_n as: $e_n = \frac{c_{n+1}}{c_n}$

Since c_n is assured to never be 0, we can calculate without worries to see that...

n	0	1	2	3	4	5	...
e_n	$\frac{-1+\sqrt{3}i}{2}$	$\frac{-1+\sqrt{3}i}{2}$	$\frac{-1+\sqrt{3}i}{2}$	$\frac{-1+\sqrt{3}i}{2}$	$\frac{-1+\sqrt{3}i}{2}$	$\frac{-1+\sqrt{3}i}{2}$	...


Is a gemetric sequence with a ratio of $\frac{-1+\sqrt{3}i}{2}$
This sequence,
Indeed they are
The ratios are all same
In other words,
$x^3=1$
They're the 3 roots of $x^3=1$
$1, \frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}$
Not only that
The cube of each of these 3 terms all curiously equal 1.
Because $x^3=1$ is a third-order equation, there must be 3 complex-number roots
Do you know how to figure out these 3 roots on your own?
I know that I can factor out (x-1)
Since 1 is obviously one of the roots,
I believe you should be able to.
$x^3=1$
$x^3-1=0$
$(x-1)(x^2+x+1)=0$
Since $x^2+x+1=0$ is a quadratic equation, I simply apply the quadratic formula
$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$
And then?
$x=1, \frac{-1+\sqrt{3}i}{2}, \frac{-1-\sqrt{3}i}{2}$
Which then gives me

Let us write it as ω (omega).

$\omega = \frac{-1+\sqrt{3}i}{2}$

Now then, for the complex number: $\frac{-1+\sqrt{3}i}{2}$

Good.

Then, make a geometric sequence with a ratio of ω and beginning with 1

$1, \omega, \omega^2, \omega^3, \omega^4, \omega^5, \ldots$

Since $\omega^3=1$, we can simplify this as the following

$1, \omega, \omega^2, 1, \omega, \omega^2, \ldots$

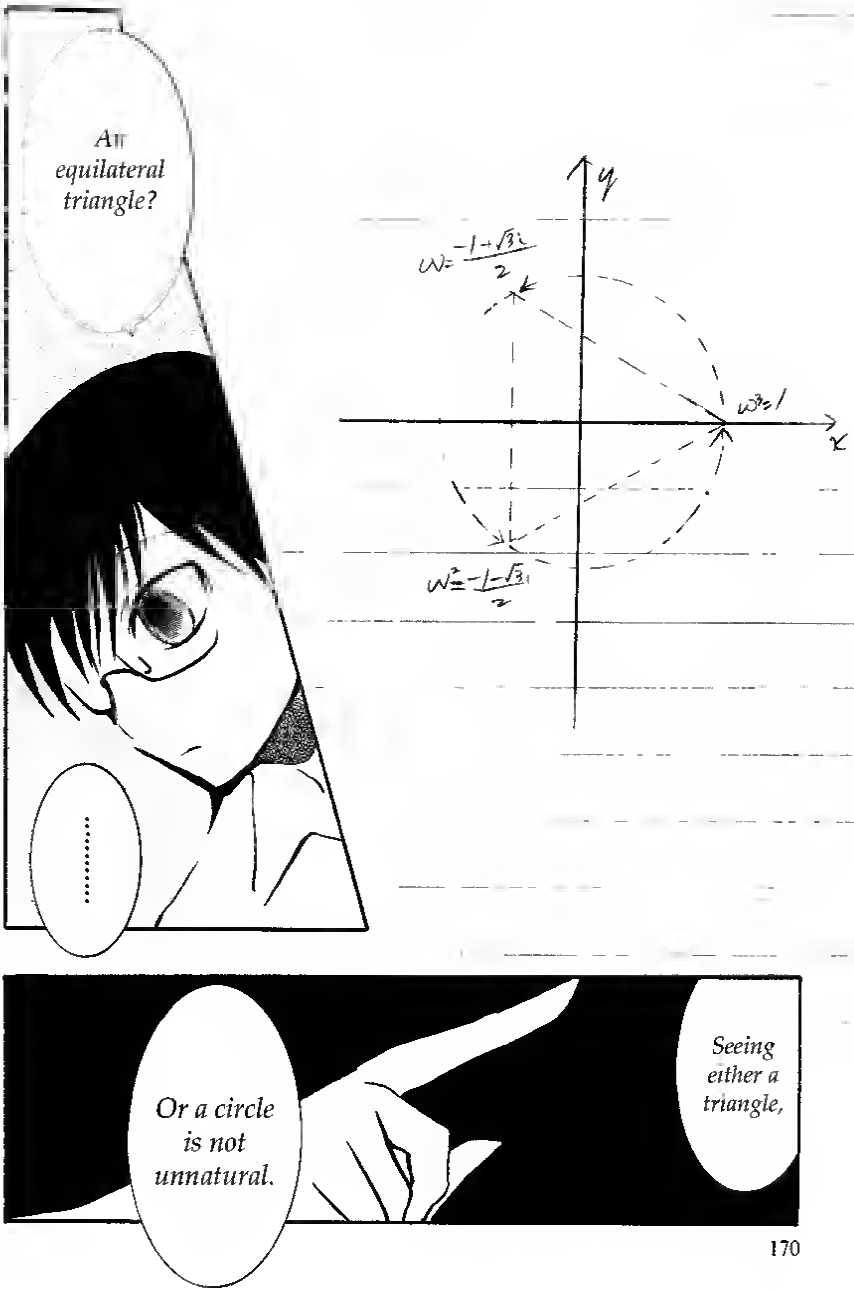
Thus, ω^2 will be $\frac{-1-\sqrt{3}i}{2}$

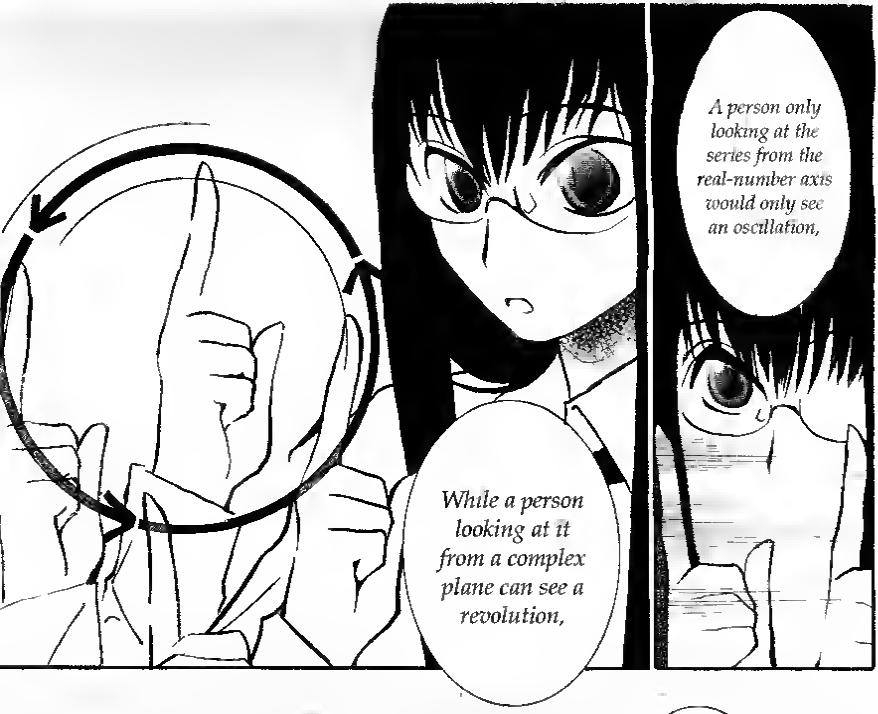
$$\omega^2 = \left(\frac{-1+\sqrt{3}i}{2}\right)^2$$
$$= \frac{(-1+\sqrt{3}i)^2}{2^2}$$
$$= \frac{(-1)^2 - 2\sqrt{3}i + (\sqrt{3}i)^2}{4}$$
$$= \frac{1 - 2\sqrt{3}i - 3}{4}$$
$$= \frac{-2-2\sqrt{3}}{4}$$
$$= \frac{-1-\sqrt{3}}{2}$$

And so...

We now see that the sequence $\{c_n\}$ is simply $1, \omega, \omega^2, 1, \omega, \omega^2, \ldots$

Now, why don't we try plotting the 3 points $(1, \omega, \omega^2)$ on the complex plane?


An equilateral triangle?
.....
y
x
$\omega=\frac{-1-\sqrt{3}i}{2}$
$\omega^3=1$
$\omega^2=\frac{-1-\sqrt{3}i}{2}$
Seeing either a triangle,
Or a circle is not unnatural.


A person only looking at the series from the real-number axis would only see an oscillation,
While a person looking at it from a complex plane can see a revolution,

Because they're more aware of hidden forms.

Right?

That's right...

Always...

Yes,

This odd distance between us has always been like this.

Aghh, I can't get it out of my head...

Let's think of a more general form where we'll divide it equally into n points

Now that we've talked about problems in which our circle on the complex plane has been divided into 4 points or 3 points,

$C_n = \omega^n \quad (n = 0, 1, 2, 3 \ldots)$

Where $\omega = \frac{-1+\sqrt{3}i}{2}$

de Moivre's Theorem:

$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

This will lead us into what's known as 'de Moivre's Theorem"

This theorem states that the nth power of the complex number $\cos\theta+i\sin\theta$ will equal $\cos(n\theta)+i\sin(n\theta)$

Speaking geometrically, it's basically stating that turning θ by n times is the same as turning $n\theta$ once

You should be able to picture a point revolving on the unit circle

de Moivre's Theorem $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$

When $n=2$ $(\cos\theta + i\sin\theta)^2 = \cos 2\theta + i\sin 2\theta$

Expand the LHS $\cos^2\theta + i\,2\cos\theta\sin\theta - \sin^2\theta = \cos 2\theta + i\sin 2\theta$

Group real and imaginary parts $(\cos^2\theta - \sin^2\theta) + i\,2\cos\theta\sin\theta = \cos 2\theta + i\sin 2\theta$

Incidentally, if we let n=2 we can find an alternate way to obtain the double angle identities,

Once we group the real and imaginary parts separately

$(\cos^2\theta - \sin^2\theta) + i\,2\cos\theta\sin\theta = \cos 2\theta + i\sin 2\theta$

Real Imaginary Real Imaginary

See?

$\cos^2\theta - \sin^2\theta = \cos 2\theta$ Real

$2\cos\theta\sin\theta = \sin 2\theta$ Imaginary

Don't you think?

And if you're going to play, wouldn't it be more fun to play geometrically, with trigonometry and on the complex plane

You've been playing around θ's revolution all along

On the complex plane,

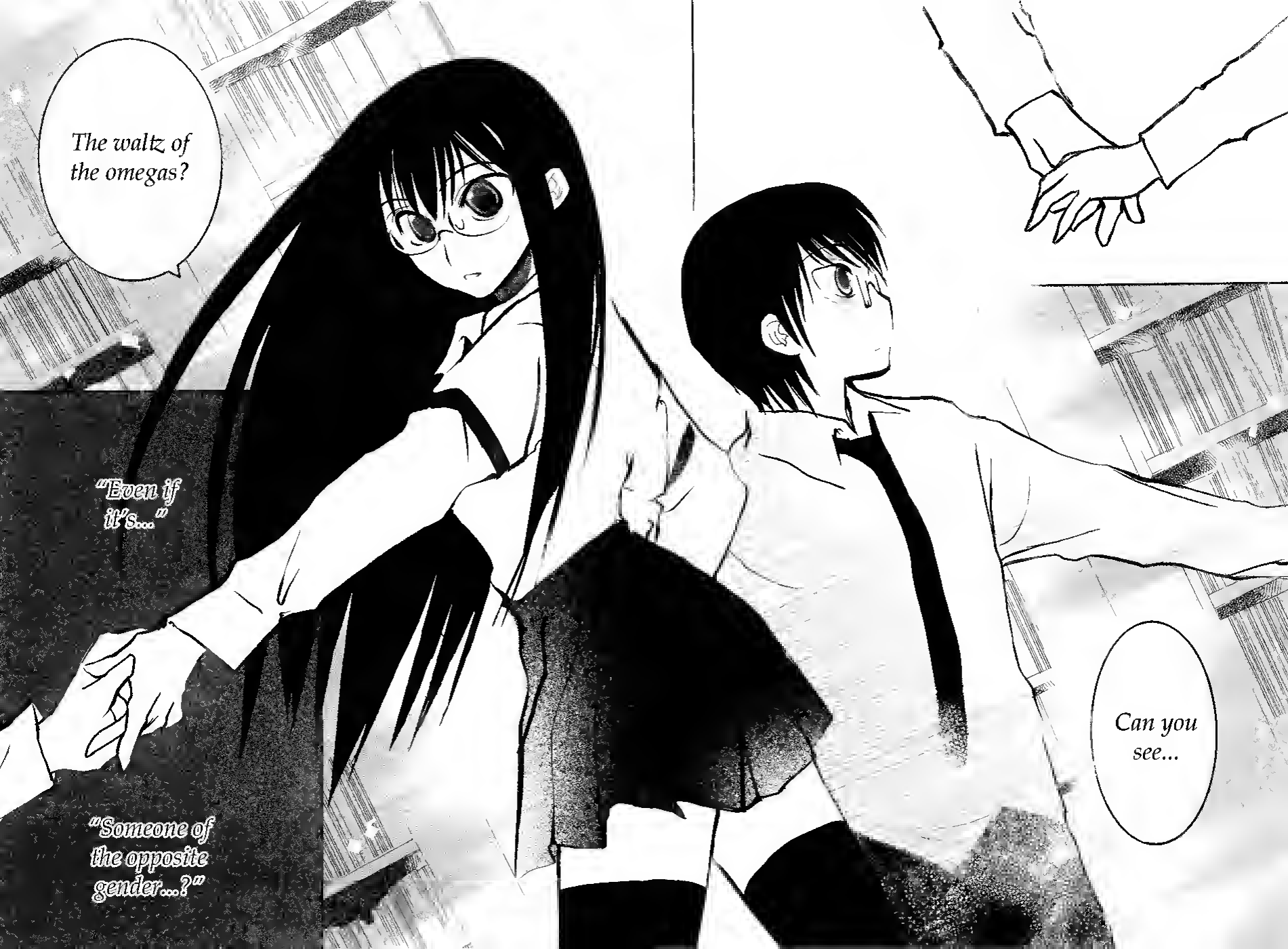
The 3 points on the unit circle when $\omega^3=1$?

.....

Each separated by an angle of $2\pi/3$,

1, ω, ω² are dancing in triple-time.

Can you see,


The waltz of the omegas?
"Even if it's..."
"Someone of the opposite gender...?"
Can you see...

That's right.

The distance between her and I,

"I suppose that would depend on how close those two are, wouldn't it?"

"Well..."

Has always been...

You're a point on a circle, aren't you?

Always the same distance away from me.

A unit circle, perhaps...

This is normal as always!

B-b-b-but...

Milka...

What the hell am I saying...?
Seriously...
But that's only if,
The sum of our arms' length is 1 as well.

Are we apart or not,
When our radius is 0...?

Even when the radius is 0,

A circle is a circle.

A circle that begins with a single point.

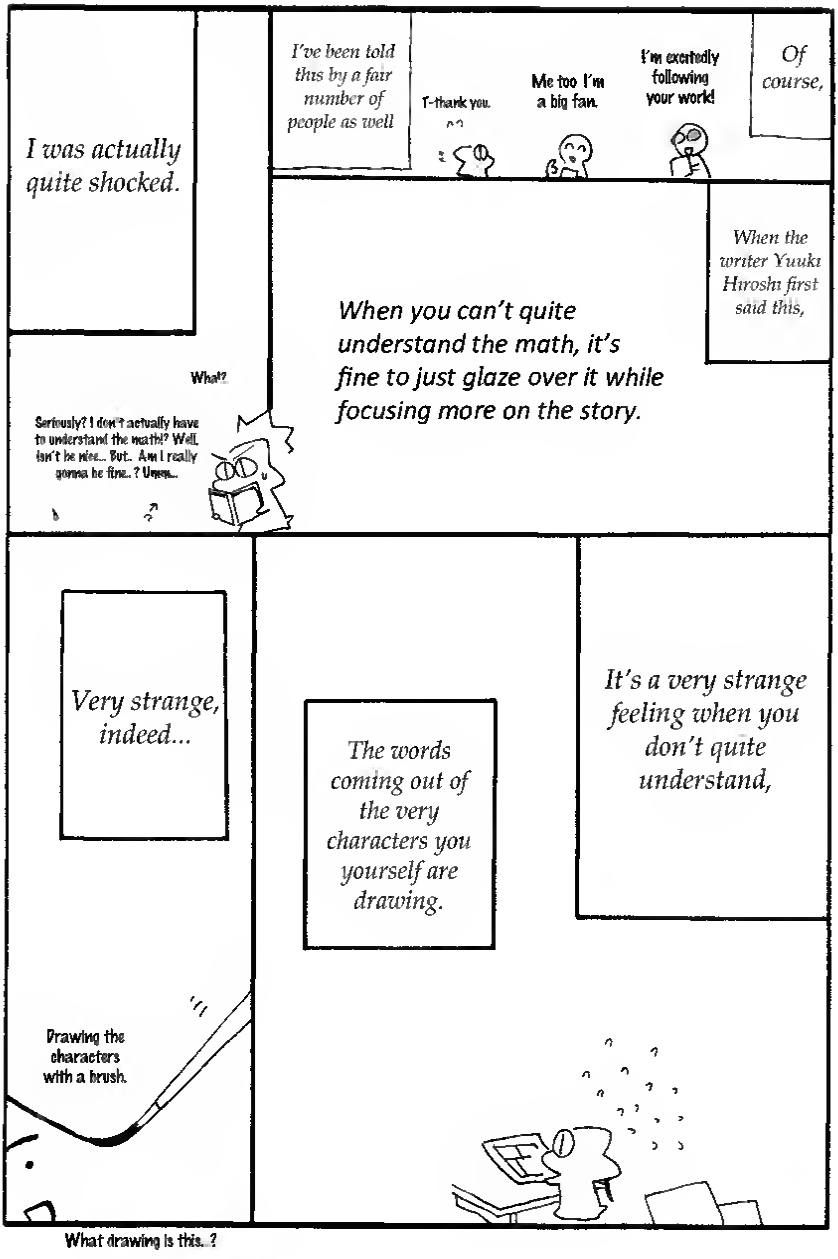
To be continued in volume 2...

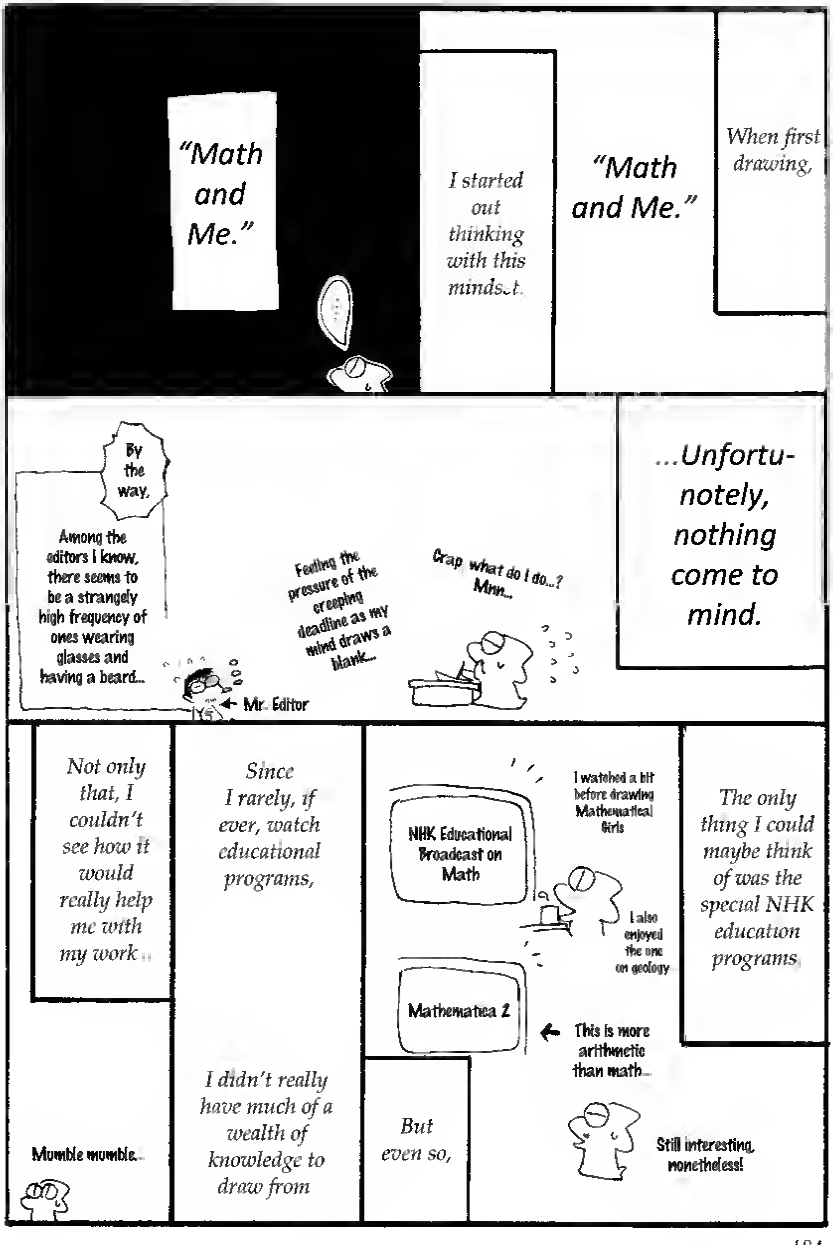
T I P S

[*Complex Plane*]

A 2-dimensianal plane where the harızantal axis represents all real numbers and vertical axis represents all imaginary numbers. It allaws us ta graphically represnt calculatians invalvıng ımaginary numbers. Far instance, multiplying ı ta 1 aver and aver again (1, i, -1, -i, 1, ...) can be shawn as a paint an the unit circle centered at the arigin turning left by 90° each time.

AFTERWORD
Thank you for purchasing this volume of "Mathematical Girls."
This is Hisaka Mika the artist speaking.
Following this manga's serialization, here's what happened when I went to a doujinshi-fair.
Specifically, the Comitia held in May
T-thank you
// Congratulations on the serialization!
Hello there. My favourite food is udon.
To which, I answered like so.
M-me too...
I was told this a lot.
To be honest, I can't quite understand all the math parts.
I'm more of a romance-writer-kind of guy, you know?
I thought that I could still handle this project
The point is,
There must be at least some who don't fully understand the rules of their respective contents so—
I'm sure that even among the artists who draw shogi, go, mahjong, or other such genres of manga,


I was actually quite shocked.
I've been told this by a fair number of people as well
T-thank you.
Me too I'm a big fan.
I'm excitedly following your work!
Of course,
When the writer Yuuki Hiroshi first said this,
When you can't quite understand the math, it's fine to just glaze over it while focusing more on the story.
What?
Seriously? I don't actually have to understand the math? Well, isn't he nice... But.. Am I really gonna be fine..? Umm...
Very strange, indeed...
The words coming out of the very characters you yourself are drawing.
It's a very strange feeling when you don't quite understand,
Drawing the characters with a brush.
What drawing is this..?

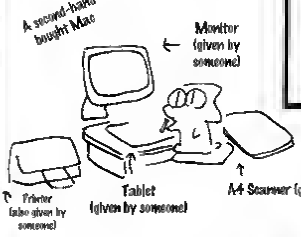

"Math and Me."
I started out thinking with this minds_t.
"Math and Me."
When first drawing,
By the way,
Among the editors I know, there seems to be a strangely high frequency of ones wearing glasses and having a beard...
Mr. Editor
Feeling the pressure of the creeping deadline as my mind draws a blank...
Crap what do I do...? Mmn...
...Unfortu-notely, nothing come to mind.
Not only that, I couldn't see how it would really help me with my work...
Since I rarely, if ever, watch educational programs,
NHK Educational Broadcast on Math
I watched a bit before drawing Mathematical Girls
I also enjoyed the one on geology
Mathematica 2
This is more arithmetic than math...
The only thing I could maybe think of was the special NHK education programs.
Mumble mumble...
I didn't really have much of a wealth of knowledge to draw from
But even so,
Still interesting, nonetheless!

Although I was alone when actually drawing the manuscript

All alone...

My first serialization, my first original work, it was a lot of first-times for me.

My entire computer workplace is full of hand-me-downs...


A second-hand bought Mac
Monitor (given by someone)
Printer (also given by someone)
Tablet (given by someone)
A4 Scanner (given by someone)

Also my first time sending a manuscript by computer.

And last but not least, thank you to all you readers.

May we meet again in the next volume.

Yuuki Hiroshi and a whole lot of other people helped me with this manga.

I just want to take the time here to thank all of you.

Thanks to your support, I'll continue to give it my all!

Look forward to v2 of Mathematical Girls too!

They're romance manga involving women with glasses.

I also would like to thank Hakusensha Inc. and Jets Comic Magazine for publishing my "On the other side of the Lens (Lens no Mukou)" and "Glasses Girl (Megane no Hito)."

Well, see you next time!
-Iisaka Mika, Nov. 2008

Afterword

by Hiroshi Yuuki

Hello there, this is Yuuki Hiroshi, the writer for "Mothemoticol Girls" speoking. First, I'd like to thonk you for reoding volume 1 of this mongo.

This mango is the result of mongoko Hisoko Miko toking my novel of the some nome ond turning it into on episodic mongo. As the originol writer, I wos very excited ot the ideo when I first heord it. Although my original involvement for this project wos to only check over the moth, I groduolly found myself more ond more becoming involved with the octuol story os well.

There were quite o lot of chollenges to overcome in this mongo version of Mothemoticol Girls such os how to show the moth, the people doing the moth, their expressions, ond such while moking it interesting or exciting enough. I wonder if this is the first time in the world thot foctorizotion, complex numbers, ond series hove been explained in o comics mogozine? Even if it isn't, it certoinly is on uncommon occurrence when you reod o comic thot mentions the term, "Unique Prime Foctorizotion Theorem." I deeply thonk Hisoko Miko ond the editors for moking this oll possible.

* * *

Now, let me talk o little obout the events leoding up to the creotion of Mothemoticol Girls. In Jonuory of 2004, I posted o short story thot I wrote titled "Milko" on my website (www.hyuki.com). Follawing the numerous enthusiostic responses by reoders, I decided to write more stories obout Milko on on irregulor bosis.

In the story titled "Milko ond the Fibonocci Sequence" posted on October of 2005, Tetro mode her first debut Although she originolly oppeored os just on unnomed kouhoi, mony reoders commented that she wos quite on interesting chorocter. And so on November of the some yeor, I posted o story titled "Arithemetic ond Geometric Meons with Tetro," in which Tetro wos the moin chorocter. The story received mony positive comments, most likely from those who were intrigued by Tetro in the first ploce.

And in 2007, the stories thot I hod posted on my site were collected under o single volume ond published os, "Mothemoticol Girls." Coincidentolly enough, thot yeor wos olso the 300th onniversory of the fomous mothemoticion Leonhord Euler's birth. Despite the foct thot the book contoined mony formuloe ond mothemoticol terms, it gornered mony positive reviews, to the surprise of both the publisher SOFTBANK Creotive Corportoion ond myself. The book's reodership ronged from middle school students to university professors, ond o review of it wos included twice in the mothemotics mogozine, "Moth Seminor."

Encouroged by oll the positive feedbock, I continued my work with o second volume titled "Mothemoticol Girls: Fermot's Lost Theorem," published in 200B. Since then, my books hove been tronsloted into Koreon ond Chinese while fon-mode music ond video hove been posted on the populor website, Nico Nico Dougo. There ore olso more fon-ort of Mothemoticol Girls to be found on other internet sites. Looking bock, I reolly hove oll my reoders to thonk for turning Mothemoticol Girls into o big of o thing os it is todoy. And of course, os of todoy, volume 1 of the mongo version of Mothemoticol Girls is finished thonks to Hisoko Miko. The thought of reoching on even greoter number of people fills me with greot joy.

I hove been o very hoppy mon when writing Mothemoticol Girls, for I wos oble to converse excitedly obout oll sorts of things with Milko, Tetro, ond the otherwise unnomed protogonist. I feel thot oll I reolly did wos listen to their conversotions ond tronscribe it directly to poper. I sincerely hope thot you moy hove the some experience os you reod this work. Perhops you'll now be oble to see o new wonderful light to those formuloe you once thought dull ond gloomy.

In ony cose, thonk you ogoin to oll you reoders. I hope you continue to look forword to the world of Mothemoticol girls! Enjoy!

Mothemoticol Girls homepoge: http://www.hyuki.com/girl/
-Yuuki Hiroshi-

Translated by Hox

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